

Name: Lois Lane

Roll Number: 14X5000π Section: B4

Note: For these multiple choice questions, no need to justify your answers.

Circle clearly the answer you choose. (You can use the back of the paper for rough work.)

Every question carries 1 mark, and only one choice (A, B, C or D) is correct.

Mark:

$\frac{5.5}{5}$

• Q.1) (Integration by parts) To (try to) compute  $\int_{-1}^1 x^2 \ln(x+1) dx$ , can we use an integration by parts?

1/1 A) Yes, it works, with  $f(x) = x^2, g'(x) = \ln(x+1)$

B) No, it fails, because  $x^2$  is not monotonic on  $[-1, 1]$

C) Yes, it works, with  $f'(x) = x^2, g(x) = \ln(x+1)$

D) No, it fails, because  $x \mapsto \ln(x+1)$  is not defined at  $x = -1$ . ✓

• Q.2) (Change of variable) Let  $a > 0, x > 0$  be two constants.

If we use the change of variable  $t = u/a$ , then the integral  $\int_0^x \frac{1}{a^2 + u^2} du$  is equal to:

1/1 A)  $\frac{1}{a} \int_0^{x/a} \frac{1}{a^2 + t^2} dt$

B)  $\frac{1}{a^2} \int_0^{ax} \frac{1}{1 + t^2} dt$

C)  $\frac{1}{a} \int_0^{x/a} \frac{1}{1 + t^2} dt$  ✓

D)  $a \int_0^{x/a} \frac{1}{1 + t} dt$

Bonus: (for +0.5 mark) With this change of variable  $t = u/a$ , conclude the computation of this integral:

+0.5 
$$\int_0^x \frac{1}{a^2 + u^2} du = ? = \frac{1}{a} \cdot \int_0^{x/a} \frac{1}{1 + t^2} dt = \frac{1}{a} \left[ \arctan(t) \right]_0^{x/a}$$
  

$$= \frac{\arctan(x/a)}{a}$$
  
 Remark:  $\arctan(t) = \tan^{-1}(t)$ .  
 Good. ✓

• Q.3) (Power functions) For  $x$  one real number such that  $x > 1$ , this expression  $x^{\frac{\ln(\ln(x))}{\ln(x)}}$  can be simplified:

1/1 A)  $x^{\frac{\ln(\ln(x))}{\ln(x)}} = \exp(x)$

B)  $x^{\frac{\ln(\ln(x))}{\ln(x)}}$  is not defined!

C)  $x^{\frac{\ln(\ln(x))}{\ln(x)}} = \ln(x)$  ✓

D)  $x^{\frac{\ln(\ln(x))}{\ln(x)}} = x$

• Q.4) (Inverse) Let  $f$  be a function defined on the interval  $(\ln(\frac{2}{5}), +\infty)$  by  $f(x) = \ln(5e^x - 2)$ .

We want to find the inverse function of  $f$ : for  $x \in D_f$  and  $y \in \mathbb{R}, y = f(x)$  if and only if  $x = f^{-1}(y)$ .

1/1 A)  $y = f(x) \iff x = f^{-1}(y) = \ln(e^y + 2) - \ln(5)$  ✓

B)  $y = f(x) \iff x = f^{-1}(y) = 5y - 2$

C)  $y = f(x) \iff x = f^{-1}(y) = \ln(5e^y - 2)$

D)  $y = f(x) \iff x = f^{-1}(y) = \ln(e^y + 5) + \ln(2)$

• Q.5) (Limits of usual functions) If  $\beta > 0$ , what is the value of this limit:  $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^\beta}$  ?

1/1 A)  $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^\beta} = -\infty$  ✓ B) that limit does not even exist!

C)  $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^\beta} = 0$

D)  $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^\beta} = 1$