

Name:

Roll Number:

Section: A__

Note: For these multiple choice questions, **no need to justify your answers.**

Circle clearly the answer you choose. (You can use the back of the paper for rough work.)

Every question carries 1 mark, and **only one choice (A, B, C or D)** is correct.

• **Q.1) (Limits of usual functions)** If $\beta > 0$, what is the value of this limit: $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^\beta}$?

- A) that limit does not even exist! B) $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^\beta} = -\infty$ C) $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^\beta} = 1$ D) $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^\beta} = 0$.

• **Q.2) (Power functions)** For x one real number such that $x > 1$, this expression $x^{\frac{\ln(\ln(x))}{\ln(x)}}$ can be simplified:

- A) $x^{\frac{\ln(\ln(x))}{\ln(x)}} = \exp(x)$ B) $x^{\frac{\ln(\ln(x))}{\ln(x)}} = x$ C) $x^{\frac{\ln(\ln(x))}{\ln(x)}} = \ln(x)$ D) $x^{\frac{\ln(\ln(x))}{\ln(x)}}$ is not defined!

• **Q.3) (Inverse)** Let f be a function defined on the interval $\left(\ln\left(\frac{2}{5}\right), +\infty\right)$ by $f(x) = \ln(5e^x - 2)$.

We want to find the inverse function of f : for $x \in D_f$ and $y \in \mathbb{R}$, $y = f(x)$ **if and only if** $x = f^{-1}(y)$.

- A) $y = f(x) \iff x = f^{-1}(y) = \ln(e^y + 2) - \ln(5)$ B) $y = f(x) \iff x = f^{-1}(y) = \ln(5e^y - 2)$
 C) $y = f(x) \iff x = f^{-1}(y) = \ln(e^y + 5) + \ln(2)$ D) $y = f(x) \iff x = f^{-1}(y) = 5y - 2$.

• **Q.4) (Change of variable)** Let $a > 0, x > 0$ be two *constants*.

If we use the change of variable $t = u/a$, then the integral $\int_0^x \frac{1}{a^2 + u^2} du$ is equal to:

- A) $\frac{1}{a^2} \int_0^{ax} \frac{1}{1+t^2} dt$ B) $\frac{1}{a} \int_0^{x/a} \frac{1}{1+t^2} dt$ C) $\frac{1}{a} \int_0^{x/a} \frac{1}{a^2+t^2} dt$ D) $a \int_0^{x/a} \frac{1}{1+t} dt$.

Bonus: (for +0.5 mark) With this change of variable $t = u/a$, conclude the computation of this integral:

$$\int_0^x \frac{1}{a^2 + u^2} du = ? =$$

• **Q.5) (Integration by parts)** To (try to) compute $\int_{-1}^1 x^2 \ln(x+1) dx$, can we use an integration by parts?

- A) Yes, it works, with $f(x) = x^2, g'(x) = \ln(x+1)$ B) No, it fails, because x^2 is not monotonic on $[-1, 1]$
 C) Yes, it works, with $f'(x) = x^2, g(x) = \ln(x+1)$ D) No, it fails, because $x \mapsto \ln(x+1)$ is not defined at $x = -1$.