

- Ex.1) Compute the TAYLOR approximation of order 4 of $x \mapsto e^x - 1 - \sin(x)$ at 0.
- Ex.2) Compute the TAYLOR approximation of order 5 of \tan at 0, first by writing this as $\frac{\sin(x)}{\cos(x)}$, and then use the classic TAYLOR-LAGRANGE identity and the fact that $\tan' = 1 + \tan^2$ to proceed by induction.
- Ex.3) Does the function $x \mapsto \sqrt{x}$ have a TAYLOR approximation of order n (for one $n \geq 1$) at 0? And at one point a for $a > 0$?
- Ex.4) Compute TAYLOR approximation of order (here and so forth abbreviated as TA) of $x \mapsto \ln(1+x)$ at order 6 at 0, thanks to the TAYLOR-YOUNG formula.
- Ex.5) Compute these TA (where $TA_n(a)$ means TAYLOR approximation of order n , for one $n \in \mathbb{N}$, at $a \in \text{Dom}(f)$):

- (a) $TA_4(0)$ of $x \mapsto \sqrt{1-x} + \sqrt{1+x}$, (c) $TA_4(0)$ of $x \mapsto \ln\left(\frac{1}{\cos(x)}\right)$,
- (b) $TA_4(0)$ of $x \mapsto (\ln(1+x))^2$, (d) $TA_3(0)$ of $x \mapsto \frac{1}{(x+1)(x+2)}$.

- Ex.6) Compute these limits :

- (a) $\lim_{x \rightarrow 0} \frac{e^x - \cos(x) - \sin(x)}{x^2}$, (d) $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$, for $a, b \in \mathbb{R}_+^*$,
- (b) $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{\ln(1+x)}$, (e) $\lim_{x \rightarrow 0} \left(\frac{a^x - b^x}{x}\right)^{\frac{1}{x}}$, for $a, b \in \mathbb{R}_+^*$.
- (c) $\lim_{x \rightarrow 0} \frac{e^{\tan(x) - \sin(x)} - 1}{x \cos(x) - \ln(1+x) - x^2/2}$,

- Ex.7) Compute *the* TA at order 2 at 0 of $x \mapsto \left(\frac{\sin(x)}{x}\right)^{\frac{3}{2}}$.

- Ex.8) Compute *the* TA at order 2 at 0 of $x \mapsto (\cos(x))^{1+\sin(x)}$.

- Ex.9) Compute *the* TA at order 2 at 0 of $x \mapsto \frac{2}{\sin^2(x)} + \frac{1}{\ln(\cos(x))}$.

- Ex.10) Study the possible prolongation at the point 0 of the function defined on $]0, 1[$ by $f(x) = \frac{1 - \cos(x)}{\tan^2(x)}$.

- Ex.11) Compute these limits :

- (i) $\lim_{x \rightarrow 0} \frac{x(e^x + 1) - 2(e^x - 1)}{x^3}$,

$$(ii) \lim_{x \rightarrow +\infty} \left(\cosh\left(\frac{1}{x}\right) \right)^{x^2},$$

$$(iii) \lim_{x \rightarrow 0} \frac{\sin(x - \sin(x))}{\sqrt{1 + x^3} - 1}.$$

Ex.12) For which $n \in \mathbb{N}$ does the function $f_n : x \mapsto |x|^n$ have a TA at order n at 0 ?