

1. Find the lengths of the paths traced ^{out} by a particle moving on a curve according to the given equation during the time interval specified in each case.

(a) $\vec{r}(t) = a(1 - \cos t)\mathbf{j} + a(t - \sin t)\mathbf{j}$
 where $0 \leq t \leq 2\pi$, $a > 0$. Ans: $8a$.

(b) $\vec{r}(t) = a \cos wt \mathbf{i} + a \sin wt \mathbf{j} + bwt \mathbf{k}$
 where $t_0 < t < t_1$, $t \in \mathbb{R}$, $t_0, t_1 \in \mathbb{R}$.
 Ans: $|w| \sqrt{a^2 + b^2} (t_1 - t_0)$

(c) $\vec{r}(t) = a \cos^3 t \mathbf{i} + a \sin^3 t \mathbf{j}$, $t \in [0, 2\pi]$
 Ans: $6a$.

(2) Find the ~~value~~ function $k(t) = \frac{\|\vec{v}(t) \times \vec{a}(t)\|}{\|\vec{v}(t)\|^3}$ for the following position vectors

(a) $\vec{r}(t) = e^t \mathbf{i} + e^{-t} \mathbf{j} + \sqrt{2}t \mathbf{k}$
 Ans: $k(t) = \sqrt{2} (e^t + e^{-t})^{-2}$

$$\vec{v}(t) = \frac{d\vec{r}}{dt}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt}$$

(b) $\vec{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$

where x and y are twice differentiable functions on I , $x, y: I \rightarrow \mathbb{R}$.

(c) $\vec{r}(t) = x \mathbf{i} + f(x) \mathbf{j}$

where $f: I \rightarrow \mathbb{R}$ is twice differentiable on I .

Ans (b): $k(t) = \frac{|x'(t)y''(t) - y'(t)x''(t)|}{\{[x'(t)]^2 + [y'(t)]^2\}^{3/2}}$

(c) $k(x) = \frac{|f''(x)|}{\{1 + [f'(x)]^2\}^{3/2}}$