

For these problems, you will study the curves with as much details as possible: domain of definition, quick drawing, asymptotes, special points, singular points, double points, extrema for  $x(t)$ ,  $y(t)$ ,  $\sqrt{x^2(t) + y^2(t)}$  etc.

Remark: this tutorial sheet is the last one for the MA101 course.

Ex.1) Study these parametric curves (be cautious on the domain of  $t$ ):

$$\begin{array}{ll} \text{(a)} \begin{cases} x(t) &= t - \sin(t) \\ y(t) &= 1 - \cos(t) \end{cases} \quad (1). & \text{(c)} \begin{cases} x(t) &= \cos^3(t) \\ y(t) &= \sin^3(t) \end{cases} \\ \text{(b)} \begin{cases} x(t) &= \frac{4t}{3+t^4} \\ y(t) &= \frac{4t^3}{\sqrt{3}3+t^4} \end{cases} & \text{(d)} \begin{cases} x(t) &= \frac{e^t}{1+t} \\ y(t) &= \frac{te^t}{1+t} \end{cases} \end{array}$$

If you are curious, you can try to look for these curves on Internet, and maybe find their names. A good website for plotting this is [www.wolframalpha.com](http://www.wolframalpha.com).

Ex.2) Study these polar curves (be cautious on the domain of  $\theta$ ):

- (a)  $\rho(\theta) = \cos(\theta) + \sin(2\theta)$ ,
- (b)  $\rho(\theta) = \cos(\theta) - \cos(2\theta)$ ,
- (c)  $\rho(\theta) = \frac{1 - 2 \cos(\theta)}{1 + \sin(\theta)}$ ,
- (d)  $\rho(\theta) = \frac{\cos(2\theta)}{2 \cos(\theta) - 1}$ ,
- (e)  $\rho(\theta) = 1 + \tan\left(\frac{\theta}{2}\right)$ , prove the existence of one *double point* on which the two tangents are orthogonal.
- (f)  $\rho(\theta) = \frac{1}{\sin^3(\theta/3)}$ , show that one line, passing to the origin  $O$  (ie the point  $(0, 0) \in \mathbb{R}^2$ ) will intersect the curve in exactly three points, and that the three tangents to the polar curve (at these points) will form a *equilateral triangle*.

Ex.3) Study the polar curve :  $\rho(\theta) = \frac{1}{\sqrt{1 + \sin(2\theta)} + \sqrt{1 - \sin(2\theta)}}$ .

First, try to do it without cheating or plotting the curve on a calculator machine or a computer.

If any interesting behavior seems to appear, try to find a way to prove it<sup>2</sup>.

<sup>1</sup>Try it on Wolfram Alpha: [www.wolframalpha.com/input/?i=plot\(x\(t\)=1-sin\(t\),y\(t\)=1-cos\(t\)\)](http://www.wolframalpha.com/input/?i=plot(x(t)=1-sin(t),y(t)=1-cos(t)))

<sup>2</sup>Here, one could try to manipulate trigonometric expressions to simplify the expression of  $\rho$  and then use Cartesian coordinates to get the final result.