

For these problems, you will study the curves with as much details as possible: domain of definition, quick drawing, asymptotes, special points, singular points, double points, extrema for $x(t)$, $y(t)$, $\sqrt{x^2(t) + y^2(t)}$ etc.

Remark: this tutorial sheet is the last one for the MA101 course.

Ex.1) Study these parametric curves (be cautious on the domain of t):

$$\begin{array}{ll} \text{(a)} \begin{cases} x(t) &= t - \sin(t) \\ y(t) &= 1 - \cos(t) \end{cases} \text{ (}^1\text{)}. & \text{(c)} \begin{cases} x(t) &= \cos^3(t) \\ y(t) &= \sin^3(t) \end{cases} \\ \text{(b)} \begin{cases} x(t) &= \frac{4t}{3 + t^4} \\ y(t) &= \frac{4}{\sqrt{3}} \frac{t^3}{3 + t^4} \end{cases} & \text{(d)} \begin{cases} x(t) &= \frac{e^t}{1 + t} \\ y(t) &= \frac{te^t}{1 + t} \end{cases} \end{array}$$

If you are curious, you can try to look for these curves on Internet, and maybe find their names. A good website for plotting this is www.wolframalpha.com.

Ex.2) Study these polar curves (be cautious on the domain of θ):

$$\text{(a)} \quad \rho(\theta) = \cos(\theta) + \sin(2\theta),$$

$$\text{(b)} \quad \rho(\theta) = \cos(\theta) - \cos(2\theta),$$

$$\text{(c)} \quad \rho(\theta) = \frac{1 - 2 \cos(\theta)}{1 + \sin(\theta)},$$

$$\text{(d)} \quad \rho(\theta) = \frac{\cos(2\theta)}{2 \cos(\theta) - 1},$$

$$\text{(e)} \quad \rho(\theta) = 1 + \tan\left(\frac{\theta}{2}\right), \text{ prove the existence of one } \textit{double point} \text{ on which the two tangents are orthogonal.}$$

$$\text{(f)} \quad \rho(\theta) = \frac{1}{\sin^3(\theta/3)}, \text{ show that one line, passing to the origin } O \text{ (ie the point } (0, 0) \in \mathbb{R}^2 \text{) will intersect the curve in exactly three points, and that the three tangents to the polar curve (at these points) will form a } \textit{equilateral triangle}.$$

Ex.3) Study the polar curve : $\rho(\theta) = \frac{1}{\sqrt{1 + \sin(2\theta)} + \sqrt{1 - \sin(2\theta)}}$.

First, try to do it without cheating or plotting the curve on a calculator machine or a computer.

If any interesting behavior seems to appear, try to find a way to prove it².

¹Try it on Wolfram Alpha: [www.wolframalpha.com/input/?i=plot\(x\(t\)=1-sin\(t\),y\(t\)=1-cos\(t\)\)](http://www.wolframalpha.com/input/?i=plot(x(t)=1-sin(t),y(t)=1-cos(t)))

²Here, one could try to manipulate trigonometric expressions to simplify the expression of ρ and then use Cartesian coordinates to get the final result.