

2. Using variation of parameters find the general solution to the following differential equations.

$$i) \quad y'' + 2y' + y = x^4 e^{-x}$$

$$ii) \quad y'' + 2y' + y = e^{-x} \log x$$

$$iii) \quad y'' - 3y' + 2y = (1 + e^{-x})^{-1}$$

Sol<sup>n</sup> i)  $y_p = \frac{1}{6} x^{-2} e^{-x}$

$$ii) \quad y_p = \frac{1}{2} x^2 e^{-x} \log x - \frac{3}{4} x^2 e^{-x}$$

$$iii) \quad y_p = e^x \log(1 + e^{-x}) - e^x + e^{2x} \log(1 + e^{-x})$$

$$iv) \quad y'' - 2y' + y = \frac{e^t}{1+t^2} + 3e^t$$

Sol<sup>n</sup>  $y_p = y_{p1} + y_{p2}$

$$y_{p1} = \frac{1}{2} e^t \ln(1+t^2) + t e^t \tan^{-1}(t)$$

$$y_{p2} = \frac{3}{2} t^2 e^t$$

$$v) \quad y'' + 3y' + 2y = \sin(e^x)$$

$$y_p = -e^{-2x} \sin(e^x)$$