

1. Let \mathbb{R} be the set of real numbers and $f : \mathbb{R} \rightarrow \mathbb{R}$ such that for all x and y in \mathbb{R}

$$|f(x) - f(y)| \leq |x - y|^3.$$

Prove that $f(x)$ is a constant function.

2. For all x in \mathbb{R} , show that between any two roots of $e^x \cos x = 1$ there exists at least one root of $e^x \sin x - 1 = 0$.