

This document is a list of all the common mistakes made by you (the students) that were seen too many times in answer scripts of the second Mid Term Exam, October 2014 at MEC.

Some of these remarks were already there in the previous document we made about mistakes in the first Mid Term Exam. We expect all of you to take note of these recurring mistakes and weaknesses, and work on them to fix those definitively.

The first and more serious issue we could point out is the following: **in any case, even in an exam when the time is limited, never start to solve a question without reading it carefully, entirely, at least twice.**

1 Common mistakes done in problem 1 (on $A(x) = \int_0^x f(t)dt$).

This problem was hard. Being able to know some of the correct implications ($a \implies \alpha$ for example) has been completely covered in the lectures, but being able to know for sure which implications is false is hard and maybe a little ahead regarding what have been covered in the tutorials. We will do our best to find more appropriate (and easy) problems for next exams.

2 Common mistakes done in problem 2 (Differentiation and Taylor's theorem).

- Again, **do not forget to introduce yourself the variable you use.** A common source of mistakes is to write $f(x)$ without mentioning the domain of definition or the set where the variable x has been taken.
- If you are using the L'Hospital rule to solve the problem, first you have to state and check the conditions for the given problem and then use the rule (it was possible to solve the question 1 $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a}$ with L'Hospital rule).
Before applying L'Hospital rule, you need to ensure applicability of that rule and need to mention the form of the given indeterminate limit (" $\frac{0}{0}$ " or " $\frac{\infty}{\infty}$ " etc).
- It was given that $a \in \mathbb{R}$ was a fixed constant, so $f(a)$ is a constant with respect to x ! Then using the linearity property of derivatives, $\frac{d}{dx}(xf(a)) = f(a)\frac{d}{dx}(x) = f(a)$.
But most of you wrote $\frac{d}{dx}(xf(a)) = xf'(a) + f(a)$ (this is wrong, because a was a fixed constant, a fixed point, not moving when x is).
- Most of you have not stated the Taylor's theorem correctly and also not in a position to identify the point $a \in \mathbb{R}$, according to the given problem.
- Some of you wrote the functions in the Taylor's series expansion, without calculating the function value and its derivative at the point $a \in \mathbb{R}$. Some of you have stated the Taylor's theorem without remainder term, then managed to prove the following result, by replacing x by c in the fifth term of the Taylor's series.

$$\forall x \in \mathbb{R}, \exists c \in (0, x) \text{ such that } 1 - \cos(2x) = 2x^2 - \frac{2}{3}\cos(2c)x^4$$

This shows that you are trying to bring the proof without knowing the remainder term in the Taylor's theorem.

- Some of you assumed (without proof) that $1 - \cos(2x) = 2x^2 - \frac{2}{3}\cos(2c)x^4$ (for any $x \in \mathbb{R}$ and one good $c \in (0, x)$), then found the limit of $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{3x^2}$.
This is better than doing nothing, but be cautious about the relationship between c and x : for any point x , there exists a c (satisfying $0 < c < x$) such that $1 - \cos(2x) = 2x^2 - \frac{2}{3}\cos(2c)x^4$.
- Remarks (and grading) for this problem was done by Prof. Satya.

3 Common mistakes done in problem 3 (on Fundamental Theorem of Calculus).

In the problem 3 especially, we saw too many of you running and trying to compute the integrals, even if the problem was only asking to find their derivatives. In particular for the first one, which was a *constant* with respect to x : no need to compute it to conclude that $\frac{d}{dx} \left(\int_{-\pi/4}^{\pi/4} \tan^2(u) du \right) = 0!$

- Not reading question properly: About 10-15% of students simply did an integration job and got zero - purely because of not reading the question.
- Neat logical exposition instead of scribbling mathematical expressions across the paper – which do not seem to relate to each other.
- Inability to write *logical, unambiguous* statements in English: Again this is across the board, and affects their performance. Early engineering education is the right time to learn to write coherent logically linked technical statements in English – and in this case there is no blame to be apportioned to your pre-engineering education.
- Remarks (and grading) for this problem was done by Prof. Arya.

4 Common mistakes done in problem 4 (on Rolle's theorem).

- Serious problems in applying basic logic. Many times the conclusion of your solution has no relation to what is said before.
- It is seen that many concluded that if $\cos(x) = e^{-x}$ then $\sin(x) = e^{-x}$ at some point x . This is same as saying $f(c) = 0$ then $f'(c) = 0$, or, equivalently, if a curve crosses the x -axis then the tangent is parallel to x -axis. Obviously this is nonsensical. Of course, $f(x) = 0$ in an interval then one can conclude $f' = 0$ in the interior of the interval. This can be carefully proven.
- When one uses Rolle's theorem (or any such theorem) one needs to show that all the conditions of the theorem such as continuity and differentiability of the function involved. At least these conditions needed to be stated for the specific function.
- As in the first exam, many use variables/functions without properly defining them.
- In part *b*) of the problem, one needs to show that a cubic can have at most three real roots by using the fact that quadratic has two roots. But some showed that if a cubic has three roots then quadratic has two roots (using Rolle's theorem). This is a correct fact but was not asked.
- Similarly, thought not relevant to the answer, many solved a quadratic equation using the standard formula for quadratic roots and concluded that there are two real roots without worrying about the sign of the determinant.

- There are two ways to solve the above problem:
 - *i*) letting n (some positive integer) be the roots of a cubic. Then using Rolle's theorem it can be concluded that there are at least $n-1$ roots for the derivative (which is a quadratic). By using the fact that quadratic can have at most two real roots we can then conclude $n-1 \leq 2$, that is, $n \leq 3$.
 - *ii*) A slight variation of the same logic is to assume that $n > 3$ and arrive at contradiction.
- Now, this logic can be easily generalized to the case of higher degree by induction.
- Remarks (and grading) for this problem was done by Prof. Vijay.

5 Common mistakes done in problem 5 (on inequalities thanks to Taylor's theorem).

- To prove a given function to be non-decreasing/increasing in an interval, you need to show that it is true at every pair of points: $\forall x, y \in I, (x < y \implies f(x) < f(y))$ and not only for $x = a, y = b$ if the function f is defined on $[a, b]$. Proving it only for one pair of points is not enough to conclude that f is increasing on $[a, b]$.
- To prove the inequalities, use the Taylor's series in remainder form with appropriate remainder term. Using the infinite Taylor series was inappropriate (this was explicitly said in the question paper: "*use the Taylor's theorem (with an explicit remainder)*").
- Using a graphical method to prove some inequality is a good idea, but *it cannot be enough*. You need to give proper mathematical proof. A drawing will help you to see what is happening and find a way to prove the result.
- A function needs to be properly defined before being used.
- Remarks (and grading) for this problem was done by Prof. Jai.

6 Common mistakes in the whole exam.

- In English, a new sentence starts with a capital letter (even in a Mathematics exam!).
- Avoid all abbreviations in an exam (especially the symbol $\&$).
- Try to keep your answer script *clean* and *readable*.
- Try to *highlight* or *underline* every result.
- Always specify which question you are solving (5.a), 1.b) etc).
- Try to "talk more", by explaining just a little bit more what you are doing and how you proceed. An answer to an entire question cannot be just maths equations, and the symbols \therefore or \therefore are not enough. However, this does not mean that one has to give long and unnecessary explanation. Just suffices to make all equations etc as part of sentences.
- Whenever new variables (not used in the question) are used, please introduce them carefully (by using statements like where $x \in \mathbb{R}$ or let x be such that...).

- Do not be confused about the difference between proving: an equality (“stuff = ... = one other thing”), an implication (“If we have P then ... then we have the result”, or $P \implies \dots \implies$ result), and an equivalence (“ P is true if and only if ... iff Q ”, or $P \Leftrightarrow \dots \Leftrightarrow Q$).
- English again: in written English, one do not use abbreviation, e.g. “we can’t” will be written “we cannot”.
- When we use binary operators ($=, \in, \subset, <, >, \leq, \geq$ etc), we try to never mix them with text: we do not write “So the minimum is > 0 ”, but “So the minimum is greater than 0” or “So $\min(E) > 0$ ”.
- Usually, it is useless and time-consuming to *write again what was the question before you start answering it*. If you start (5.a), do not loose time by writing “We have to show that $(f(x))^2 + (g(x))^2 = 1, \forall x \in \mathbb{R}$ ”. Just start. We know what is the question, and you also know because you have the question paper just in front in you.
- English again: try to not forget to add a “n” at the end of the pronoun “a” before another word starting with a vowel. We write “Let a be an element of E ”, “an upper bound”, “an even function” etc.