

Problem: Fill in the blanks.

Theorem: Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differential function and f has a local minimum at $x \in \mathbb{R}$. Then $f'(x) = \underline{\hspace{2cm}}$ and $f''(x) \underline{\hspace{2cm}}$.

Proof: We will follow the following steps.

- Since x is a local minimum there exists a $\underline{\hspace{2cm}}$ number $\delta \in \mathbb{R}$ such that

$$f(x) \leq f(x+h), \quad h \in (\underline{\hspace{1cm}}, \underline{\hspace{1cm}}). \quad (1)$$

- Hence, it follows from (1) that

$$\frac{f(x+h) - f(x)}{h} \geq 0, \quad h \in (0, \underline{\hspace{1cm}}), \quad (2)$$

$$\frac{f(x+h) - f(x)}{h} \leq 0, \quad h \in (\underline{\hspace{1cm}}, 0). \quad (3)$$

- Now, (2) and (3) imply that

$$\lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h} \underline{\hspace{1cm}} 0, \quad (4)$$

$$\lim_{h \rightarrow 0^-} \frac{f(x+h) - f(x)}{h} \underline{\hspace{1cm}} 0. \quad (5)$$

- Now, since f is differentiable everywhere in \mathbb{R} it follows from (4) and (5) that

$$\lim_{h \rightarrow 0^-} \frac{f(x+h) - f(x)}{h} \underline{\hspace{1cm}} \lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h} \underline{\hspace{1cm}} 0, \quad (6)$$

which further implies that $f'(x) \underline{\hspace{2cm}}$.