

Problem 3

1. For the following Initial Value Problem (IVP) of order 1 :

$$y' + \frac{2}{x}y = \frac{\cos(x)}{x^2}, \quad y(\pi) = 0.$$

2. Find the largest interval in which the solution of the IVP is certain to exist and to be unique.
3. Find the solutions of the homogeneous equation $y' + \frac{2}{x}y = 0$ on that interval.
4. Then, by using the method of variation of parameter, find the solutions of the complete equation $y' + \frac{2}{x}y = \frac{\cos(x)}{x^2}$.
5. Quickly but properly check that the functions you give as solutions are indeed solutions, by the method you like.