CoursMagistral_4-5

October 18, 2019

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2 ALGO1 : Introduction à l'algorithmique

- Page du cours : https://perso.crans.org/besson/teach/info1_algo1_2019/
- Magistère d'Informatique de Rennes - ENS Rennes - Année 2019/2020
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  - Travaux dirigés : Raphaël Truffet

- Références :
  - Open Data Structures

3 Cours Magistral 4 & 5

- Ce cours traite de graphes.
- On donne le type abstrait des graphes, et plusieurs implémentations de la même structure de données (plusieurs classes).
- CM4 : On implémente le parcours en profondeur, qu'on illustre sur quelques exemples.
- CM5 : On implémente le parcours en largeur, qu'on illustre sur quelques exemples.

3.1 Type abstrait des α graphes

On se donne un type α pour les sommets, et on va en fait se restreindre à $\alpha =$ `int`, et les sommets seront $\{0, \dots, n-1\}$ où $n = |S|$ pour des graphes $G = (S, A)$.

On va écrire une classe qui implémente des opérations "plus haut niveau", en fonction des opérations bas niveau.

Pour l'afficher, on va utiliser la librarie networkx

```
In [140]: import networkx as nx
```

```
In [170]: class BaseGraph():
              def out_degree(self, vertex):
                  return len(self.succ(vertex))

              def in_degree(self, vertex):
                  return len(self.pred(vertex))

              def degree(self, vertex):
                  return len(self.neighbors(vertex))

              def is_vertex(self, vertex):
                  """ Test presence of a vertex."""
                  return vertex in self.vertexes

              @property
              def vertexes(self):
                  """ List of vertexes."""
                  return list(range(self.nb_vertexes))

              def is_neighbor(self, u, v):
```

```
        """ Test neighborhood."""
        return u in self.neighbors(v)

    @property
    def edges(self):
        """ Set of edges (pairs), in O(|A|) if well implemented."""
        return {(u, v) for u in self.vertexes for v in self.neighbors(u)}

    @property
    def nb_edges(self):
        return len(self.edges)

    def draw(self):
        G = nx.DiGraph() if self.oriented else nx.Graph()
        G.add_nodes_from(self.vertexes)
        G.add_edges_from(self.edges)
        return nx.draw_kamada_kawai(G, with_labels=True, font_weight='bold')
```

3.1.1 Un premier exemple de graphe

On va travailler avec le graphe exemple suivant, qu'il soit orienté ou non :

Orienté	Non orienté

```
In [183]: def defaultGraph(GraphClass, oriented=True):
              print(f"Creating empty graph with class {GraphClass}...")
              graph = GraphClass(oriented=oriented)
              n = 7
              for i in range(n):
                  print(f"Adding vertex {i}...")
                  graph.add_vertex(i)
              for edge in [(0, 1), (0, 2), (1, 3), (1, 4), (2, 5), (2, 6)]:
                  print(f"Adding edge {edge}...")
                  graph.add_edge(*edge)
              return graph
```

```
In [184]: plt.figure()
          defaultGraph(AdjMatrixGraph, oriented=True).draw()
          plt.figure()
          defaultGraph(AdjMatrixGraph, oriented=False).draw()
```

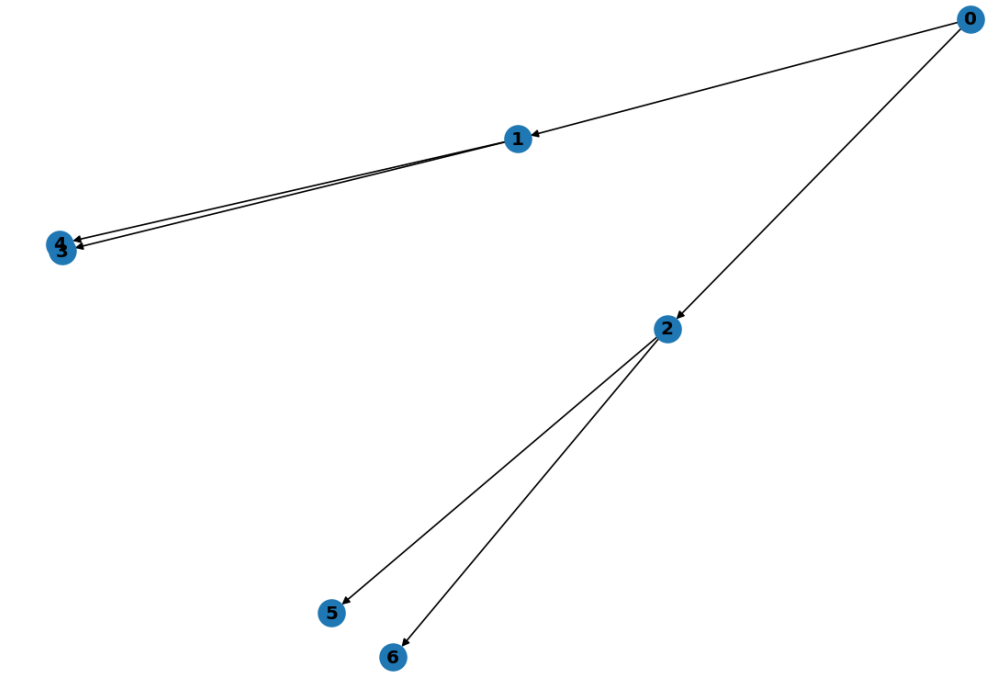
```
Out[184]: <Figure size 1200x840 with 0 Axes>
```

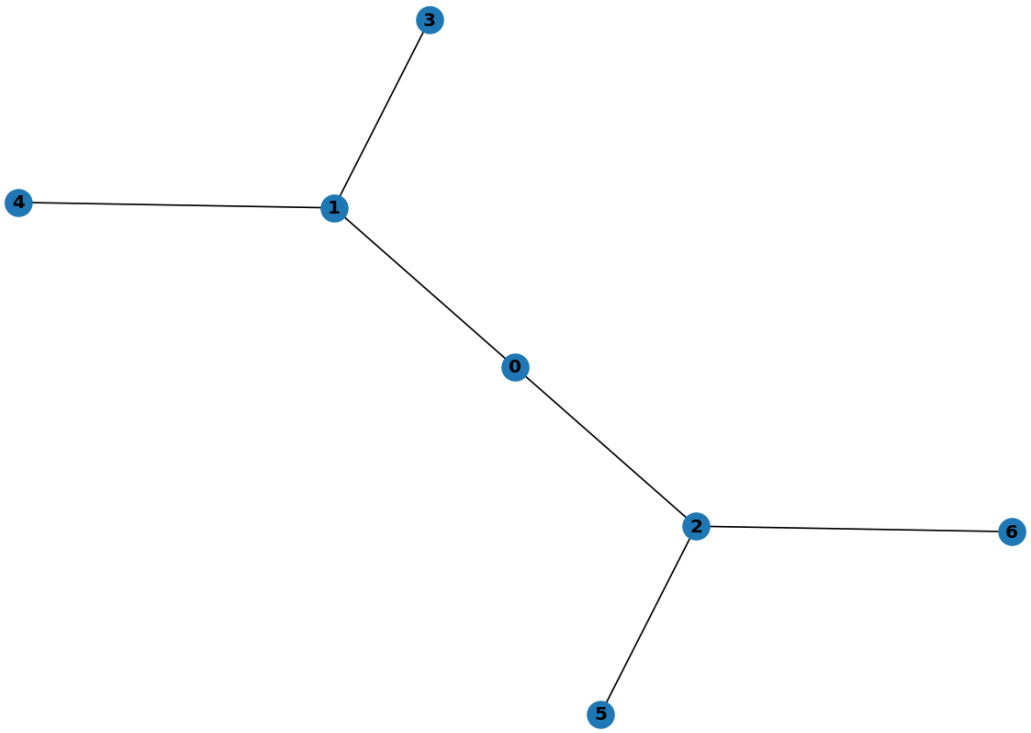
```
Creating empty graph with class <class '__main__.AdjMatrixGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
```

```
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
Adding edge (2, 6)...
```

```
Out[184]: <Figure size 1200x840 with 0 Axes>
```

```
Creating empty graph with class <class '__main__.AdjMatrixGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
Adding edge (2, 6)...
```


0
1
4
3
2
5
6


3
4
1
0
2
6
5

On va tester les différentes implémentations avec la petite fonction suivante, qui vérifie que l'on peut accéder à toute l'information contenu dans le graphe.

```
In [143]: def test_defaultGraph(GraphClass):
              for oriented in [True, False]:
                  graph = defaultGraph(GraphClass, oriented=oriented)
                  print(f"Graph:")
                  print(graph)
                  print(f"Number of vertexes: {graph.nb_vertexes}")
                  print(f"Is the graph oriented? {graph.oriented}")
                  print(f"Number of edges: {graph.nb_edges}")
                  print(f"List of vertexes: {graph.vertexes}")
                  print(f"List of edges: {graph.edges}")
                  for i in graph.vertexes:
                      print(f"  List of neighbors of {i}: {graph.neighbors(i)} (degree {graph.
                      print(f"    List of succ of {i}: {graph.succ(i)} (out degree {graph.out_
                      print(f"    List of pred of {i}: {graph.pred(i)} (in degree {graph.in_deg
```

3.1.2 Graphe aléatoire de taille n

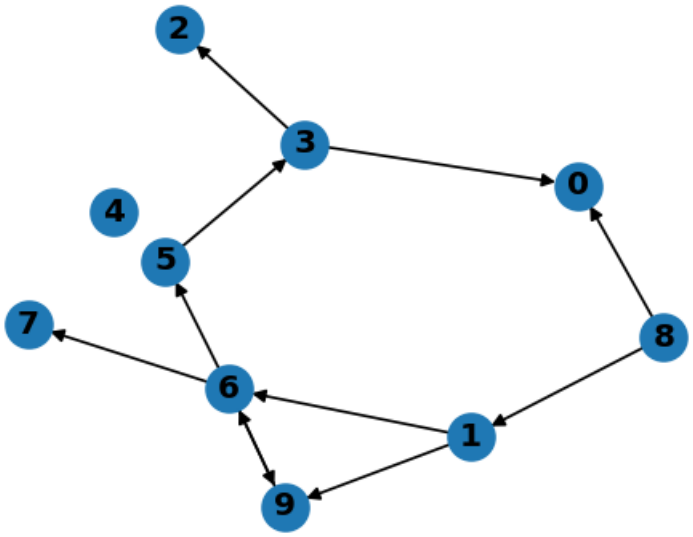
On va étudier un graphe aléatoire suivant un modèle très simple : on fixe n le nombre de sommets, et ensuite chaque arête (i, j) est ajoutée dans le graphe avec une probabilité $p \in (0, 1)$ fixée (graphe d'Erdös-Rényi).

```
In [144]: import random

In [145]: def with_probability(p):
              return random.random() <= p

In [146]: def randomGraph(GraphClass, n=10, probability=0.1, oriented=True):
              graph = GraphClass(oriented=oriented)
              for i in range(n):
                  graph.add_vertex(i)
              for i in range(n):
                  for j in range(n):
                      if with_probability(probability):
                          graph.add_edge(i, j)
              return graph

In [147]: print(randomGraph(AdjMatrixGraph, 10, 0.1, oriented=True))

[[False False False False False False False False False False]
 [False False  True False False False  True False False False]
 [False False False False False False False False False False]
 [False False False False False False False False False False]
 [False False False  True False False False False False False]
 [False False False False False False False False False False]
```

```
 [False False False False False False  True False False False]
 [False False False False False  True False False False False]
 [False False False False False False False False False False]
 [False False False False False False False False False False]]
```

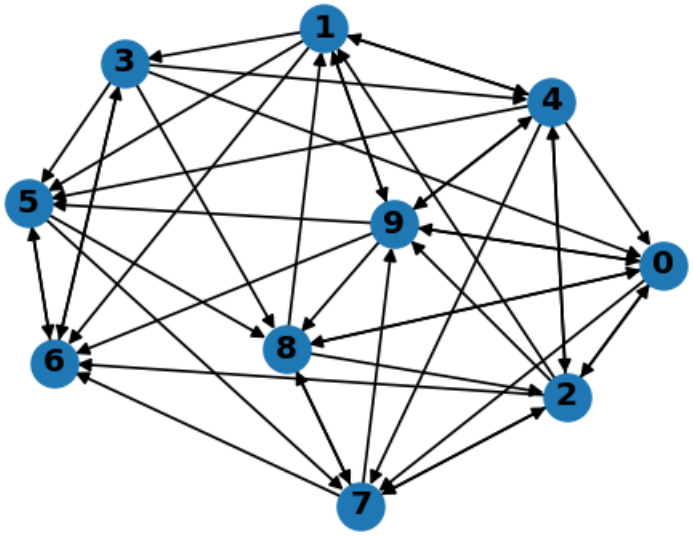
```
In [189]: plt.figure(figsize=(4, 3))
          randomGraph(AdjMatrixGraph, 10, 0.1, oriented=True).draw()
```

```
Out[189]: <Figure size 480x360 with 0 Axes>
```



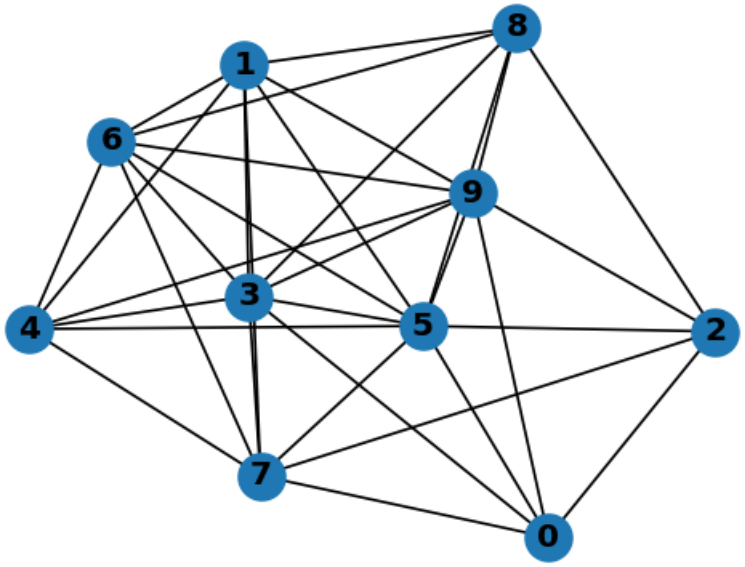

```
In [190]: plt.figure(figsize=(4, 3))
          randomGraph(AdjMatrixGraph, 10, 0.5, oriented=True).draw()
```

```
Out[190]: <Figure size 480x360 with 0 Axes>
```



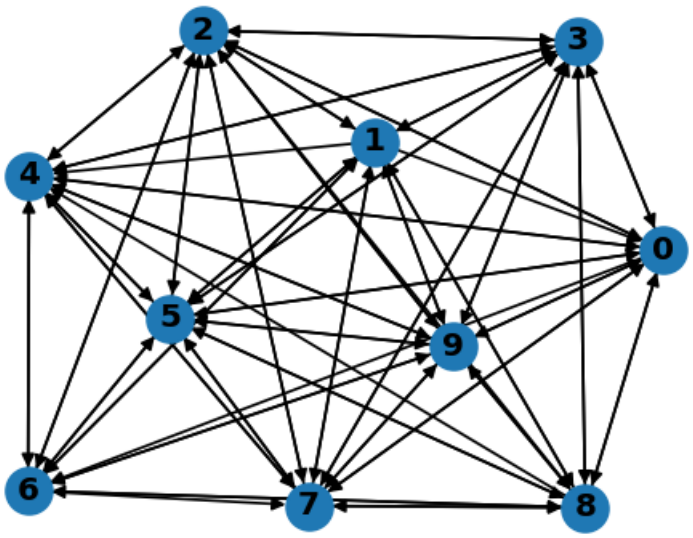

```
In [191]: plt.figure(figsize=(4, 3))
          randomGraph(AdjMatrixGraph, 10, 0.5, oriented=False).draw()
```

```
Out[191]: <Figure size 480x360 with 0 Axes>
```




```
In [192]: plt.figure(figsize=(4, 3))
          randomGraph(AdjMatrixGraph, 10, 0.9, oriented=True).draw()
```

Out[192]: <Figure size 480x360 with 0 Axes>



3.2 Trois différentes implémentations

3.2.1 Graphes par matrice d'adjacence

```
In [173]: import numpy as np
```

```
In [174]: class AdjMatrixGraph(BaseGraph):
              def __init__(self, oriented=True, n=0):
                  """ Takes O(n^2) time and space."""
                  self.oriented = oriented
                  self.nb_vertexes = n
                  self._matrix = np.zeros((n, n), dtype=bool)

              def __str__(self):
                  return str(self._matrix)

              def is_vertex(self, vertex):
                  """ Test presence of a vertex."""
                  return 0 <= vertex < self.nb_vertexes

              def add_vertex(self, m):
                  """ Worst case is O(m^2) to extend the matrix. Best case is O(1) if nothing
                  if not self.is_vertex(m):
                      n = self.nb_vertexes
```

```
            assert 0 <= m and n <= m
            self.nb_vertexes = m + 1
            old_matrix = self._matrix[:,:]
            # extend the matrix
            self._matrix = np.zeros((m + 1, m + 1), dtype=bool)
            # copy the old matrix
            self._matrix[:n, :n] = old_matrix

    def add_edge(self, i, j):
        """ O(1) time."""
        assert 0 <= i < self.nb_vertexes and 0 <= j < self.nb_vertexes
        self._matrix[i, j] = True
        if not self.oriented:
            self._matrix[j, i] = True

    def remove_vertex(self, m):
        """ TODO do it yourself, it's not hard!"""
        raise NotImplementedError

    def remove_edge(self, i, j):
        """ O(1) time."""
        assert 0 <= i < self.nb_vertexes and 0 <= j < self.nb_vertexes
        self._matrix[i, j] = False
        if not self.oriented:
            self._matrix[j, i] = False

    def merge_vertexes(self, i, j):
        """ TODO do it yourself, it's not hard!"""
        raise NotImplementedError

    def pred(self, i):
        assert 0 <= i < self.nb_vertexes
        return [j for j in self.vertexes if self.is_neighbor(j, i)]

    def is_pred(self, u, v):
        """ O(1) time."""
        return self._matrix[v, u]

    def succ(self, i):
        assert 0 <= i < self.nb_vertexes
        return [j for j in self.vertexes if self.is_neighbor(i, j)]

    def is_succ(self, u, v):
        """ O(1) time."""
        return self._matrix[u, v]

    def neighbors(self, i):
        assert 0 <= i < self.nb_vertexes
```

```
            if self.oriented:
                return self.succ(i)
            else:
                return [j for j in self.vertexes if self.is_neighbor(i, j) or self.is_ne

        def is_neighbor(self, u, v):
            """ O(1) time."""
            if self.oriented:
                return self._matrix[u, v]
            else:
                return self._matrix[u, v] or self._matrix[v, u]
```

Testons cette première implémentation :

```
In [175]: defaultGraph(AdjMatrixGraph)

Creating empty graph with class <class '__main__.AdjMatrixGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
Adding edge (2, 6)...


Out[175]: <__main__.AdjMatrixGraph at 0x7f395ccfe5c0>

In [176]: test_defaultGraph(AdjMatrixGraph)

Creating empty graph with class <class '__main__.AdjMatrixGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
```

```
Adding edge (2, 6)...
Graph:
[[False  True  True False False False False]
 [False False False  True  True False False]
 [False False False False False  True  True]
 [False False False False False False False]
 [False False False False False False False]
 [False False False False False False False]
 [False False False False False False False]]
Number of vertexes: 7
Is the graph oriented? True
Number of edges: 6
List of vertexes: [0, 1, 2, 3, 4, 5, 6]
List of edges: {(0, 1), (1, 3), (2, 6), (1, 4), (2, 5), (0, 2)}
  List of neighbors of 0: [1, 2] (degree 2)
    List of succ of 0: [1, 2] (out degree 2)
    List of pred of 0: [] (in degree 0)
  List of neighbors of 1: [3, 4] (degree 2)
    List of succ of 1: [3, 4] (out degree 2)
    List of pred of 1: [0] (in degree 1)
  List of neighbors of 2: [5, 6] (degree 2)
    List of succ of 2: [5, 6] (out degree 2)
    List of pred of 2: [0] (in degree 1)
  List of neighbors of 3: [] (degree 0)
    List of succ of 3: [] (out degree 0)
    List of pred of 3: [1] (in degree 1)
  List of neighbors of 4: [] (degree 0)
    List of succ of 4: [] (out degree 0)
    List of pred of 4: [1] (in degree 1)
  List of neighbors of 5: [] (degree 0)
    List of succ of 5: [] (out degree 0)
    List of pred of 5: [2] (in degree 1)
  List of neighbors of 6: [] (degree 0)
    List of succ of 6: [] (out degree 0)
    List of pred of 6: [2] (in degree 1)
Creating empty graph with class <class '__main__.AdjMatrixGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
```

```
Adding edge (2, 6)...
Graph:
[[False  True  True False False False False]
 [ True False False  True  True False False]
 [ True False False False False  True  True]
 [False  True False False False False False]
 [False  True False False False False False]
 [False False  True False False False False]
 [False False  True False False False False]]
Number of vertexes: 7
Is the graph oriented? False
Number of edges: 12
List of vertexes: [0, 1, 2, 3, 4, 5, 6]
List of edges: {(0, 1), (1, 3), (2, 6), (3, 1), (5, 2), (1, 4), (2, 0), (2, 5), (6, 2), (1, 0)
  List of neighbors of 0: [1, 2] (degree 2)
    List of succ of 0: [1, 2] (out degree 2)
    List of pred of 0: [1, 2] (in degree 2)
  List of neighbors of 1: [0, 3, 4] (degree 3)
    List of succ of 1: [0, 3, 4] (out degree 3)
    List of pred of 1: [0, 3, 4] (in degree 3)
  List of neighbors of 2: [0, 5, 6] (degree 3)
    List of succ of 2: [0, 5, 6] (out degree 3)
    List of pred of 2: [0, 5, 6] (in degree 3)
  List of neighbors of 3: [1] (degree 1)
    List of succ of 3: [1] (out degree 1)
    List of pred of 3: [1] (in degree 1)
  List of neighbors of 4: [1] (degree 1)
    List of succ of 4: [1] (out degree 1)
    List of pred of 4: [1] (in degree 1)
  List of neighbors of 5: [2] (degree 1)
    List of succ of 5: [2] (out degree 1)
    List of pred of 5: [2] (in degree 1)
  List of neighbors of 6: [2] (degree 1)
    List of succ of 6: [2] (out degree 1)
    List of pred of 6: [2] (in degree 1)
```

3.2.2 Graphes par listes d'adjacence

```
In [177]: class AdjListsGraph(BaseGraph):
              def __init__(self, oriented=True, n=0):
                  """ Takes O(n) time and space to allocate the empty lists."""
                  self.oriented = oriented
                  self.nb_vertexes = n
                  self._lists = [ [] for i in range(n) ]

              def __str__(self):
                  return str(self._lists)
```

```
    def is_vertex(self, vertex):
        """ Test presence of a vertex."""
        return 0 <= vertex < self.nb_vertexes

    def add_vertex(self, m):
        """ Worst case is O(m^2) to extend the matrix. Best case is O(1) if nothing
        if not self.is_vertex(m):
            n = self.nb_vertexes
            assert 0 <= m and n <= m
            self.nb_vertexes = m + 1
            self._lists = [ [] if i >= n else self._lists[i] for i in range(m + 1) ]

    def add_edge(self, i, j):
        """ O(1) time: append j in head of list of neighbors of i."""
        assert 0 <= i < self.nb_vertexes and 0 <= j < self.nb_vertexes
        if not self.is_neighbor(i, j):
            self._lists[i].append(j)
        if not self.oriented:
            if not self.is_neighbor(j, i):
                self._lists[j].append(i)

    def remove_vertex(self, m):
        """ TODO do it yourself, it's not hard!"""
        raise NotImplementedError

    def remove_edge(self, i, j):
        """ O(1) time."""
        assert 0 <= i < self.nb_vertexes and 0 <= j < self.nb_vertexes
        self._lists[i].remove(j)
        if not self.oriented:
            self._lists[j].remove(i)

    def merge_vertexes(self, i, j):
        """ TODO do it yourself, it's not hard!"""
        raise NotImplementedError

    def pred(self, i):
        """ Not trivial, has to check all lists, in O(|S|*|A|)."""
        assert 0 <= i < self.nb_vertexes
        return [j for j in self.vertexes if self.is_pred(j, i)]

    def is_pred(self, u, v):
        """ O(|S|) time in worst case."""
        return u in self._lists[v]

    def succ(self, i):
        """ Create a new list, to be sure that we don't modify the underlying self._
```

```
            assert 0 <= i < self.nb_vertexes
            return list(self._lists[i])

        def is_succ(self, u, v):
            """ O(|S|) time in worst case."""
            return v in self._lists[u]

        def neighbors(self, i):
            assert 0 <= i < self.nb_vertexes
            if self.oriented:
                return list(self.succ(i))
            else:
                return [j for j in self.vertexes if self.is_neighbor(i, j)]

        def is_neighbor(self, u, v):
            """ O(|S|) time in worst case."""
            if self.oriented:
                return v in self._lists[u]
            else:
                return v in self._lists[u] or u in self._lists[v]
```

Testons cette première implémentation :

```
In [178]: defaultGraph(AdjListsGraph)

Creating empty graph with class <class '__main__.AdjListsGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
Adding edge (2, 6)...


Out[178]: <__main__.AdjListsGraph at 0x7f395cb578d0>

In [179]: test_defaultGraph(AdjListsGraph)

Creating empty graph with class <class '__main__.AdjListsGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
```

```
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
Adding edge (2, 6)...
Graph:
[[1, 2], [3, 4], [5, 6], [], [], [], []]
Number of vertexes: 7
Is the graph oriented? True
Number of edges: 6
List of vertexes: [0, 1, 2, 3, 4, 5, 6]
List of edges: {(0, 1), (1, 3), (2, 6), (1, 4), (2, 5), (0, 2)}
  List of neighbors of 0: [1, 2] (degree 2)
    List of succ of 0: [1, 2] (out degree 2)
    List of pred of 0: [1, 2] (in degree 2)
  List of neighbors of 1: [3, 4] (degree 2)
    List of succ of 1: [3, 4] (out degree 2)
    List of pred of 1: [3, 4] (in degree 2)
  List of neighbors of 2: [5, 6] (degree 2)
    List of succ of 2: [5, 6] (out degree 2)
    List of pred of 2: [5, 6] (in degree 2)
  List of neighbors of 3: [] (degree 0)
    List of succ of 3: [] (out degree 0)
    List of pred of 3: [] (in degree 0)
  List of neighbors of 4: [] (degree 0)
    List of succ of 4: [] (out degree 0)
    List of pred of 4: [] (in degree 0)
  List of neighbors of 5: [] (degree 0)
    List of succ of 5: [] (out degree 0)
    List of pred of 5: [] (in degree 0)
  List of neighbors of 6: [] (degree 0)
    List of succ of 6: [] (out degree 0)
    List of pred of 6: [] (in degree 0)
Creating empty graph with class <class '__main__.AdjListsGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
```

```
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
Adding edge (2, 6)...
Graph:
[[1, 2], [3, 4], [5, 6], [], [], [], []]
Number of vertexes: 7
Is the graph oriented? False
Number of edges: 12
List of vertexes: [0, 1, 2, 3, 4, 5, 6]
List of edges: {(0, 1), (1, 3), (2, 6), (3, 1), (5, 2), (1, 4), (2, 0), (2, 5), (6, 2), (1, 0)
  List of neighbors of 0: [1, 2] (degree 2)
    List of succ of 0: [1, 2] (out degree 2)
    List of pred of 0: [1, 2] (in degree 2)
  List of neighbors of 1: [0, 3, 4] (degree 3)
    List of succ of 1: [3, 4] (out degree 2)
    List of pred of 1: [3, 4] (in degree 2)
  List of neighbors of 2: [0, 5, 6] (degree 3)
    List of succ of 2: [5, 6] (out degree 2)
    List of pred of 2: [5, 6] (in degree 2)
  List of neighbors of 3: [1] (degree 1)
    List of succ of 3: [] (out degree 0)
    List of pred of 3: [] (in degree 0)
  List of neighbors of 4: [1] (degree 1)
    List of succ of 4: [] (out degree 0)
    List of pred of 4: [] (in degree 0)
  List of neighbors of 5: [2] (degree 1)
    List of succ of 5: [] (out degree 0)
    List of pred of 5: [] (in degree 0)
  List of neighbors of 6: [2] (degree 1)
    List of succ of 6: [] (out degree 0)
    List of pred of 6: [] (in degree 0)
```

3.2.3 Graphes par liste d'arêtes

```
In [180]: class EdgesListGraph(BaseGraph):
              def __init__(self, oriented=True, n=0):
                  """ Takes O(n) time and space to allocate the empty lists."""
                  self.oriented = oriented
                  self.nb_vertexes = n
                  self._edges = set()

              def __str__(self):
                  return str(self._edges)

              def is_vertex(self, vertex):
                  """ Test presence of a vertex."""
```

```
        return 0 <= vertex < self.nb_vertexes

    def add_vertex(self, m):
        """ Worst case is O(m^2) to extend the matrix. Best case is O(1) if nothing
        if not self.is_vertex(m):
            n = self.nb_vertexes
            assert 0 <= m and n <= m
            self.nb_vertexes = m + 1

    def add_edge(self, i, j):
        """ O(1) time: append j in head of list of neighbors of i."""
        assert 0 <= i < self.nb_vertexes and 0 <= j < self.nb_vertexes
        if not self.is_neighbor(i, j):
            self._edges.add((i, j))
        if not self.oriented:
            if not self.is_neighbor(j, i):
                self._edges.add((j, i))

    def remove_vertex(self, m):
        """ TODO do it yourself, it's not hard!"""
        raise NotImplementedError

    def remove_edge(self, i, j):
        """ O(1) time."""
        assert 0 <= i < self.nb_vertexes and 0 <= j < self.nb_vertexes
        self._edges.remove((i, j))
        if not self.oriented:
            self._edges.remove((j, i))

    def merge_vertexes(self, i, j):
        """ TODO do it yourself, it's not hard!"""
        raise NotImplementedError

    def pred(self, i):
        assert 0 <= i < self.nb_vertexes
        return [j for j in self.vertexes if self.is_pred(j, i)]

    def is_pred(self, u, v):
        """ O(|S|) time in worst case."""
        return (v, u) in self._edges

    def succ(self, i):
        assert 0 <= i < self.nb_vertexes
        return [j for j in self.vertexes if self.is_succ(i, j)]

    def is_succ(self, u, v):
        """ O(|S|) time in worst case."""
        return (u, v) in self._edges
```

```
    def neighbors(self, i):
        assert 0 <= i < self.nb_vertexes
        return [j for j in self.vertexes if self.is_neighbor(j, i)]

    def is_neighbor(self, u, v):
        """ O(|S|) time in worst case."""
        if self.oriented:
            return (u, v) in self._edges
        else:
            return (u, v) in self._edges or (v, u) in self._edges
```

Testons cette première implémentation :

```
In [181]: defaultGraph(EdgesListGraph)

Creating empty graph with class <class '__main__.EdgesListGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
Adding edge (2, 6)...


Out[181]: <__main__.EdgesListGraph at 0x7f394b11bcc0>

In [182]: test_defaultGraph(EdgesListGraph)

Creating empty graph with class <class '__main__.EdgesListGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
```

```
Adding edge (2, 6)...
Graph:
{(0, 1), (1, 3), (2, 6), (1, 4), (2, 5), (0, 2)}
Number of vertexes: 7
Is the graph oriented? True
Number of edges: 6
List of vertexes: [0, 1, 2, 3, 4, 5, 6]
List of edges: {(3, 1), (5, 2), (2, 0), (6, 2), (1, 0), (4, 1)}
  List of neighbors of 0: [] (degree 0)
    List of succ of 0: [1, 2] (out degree 2)
    List of pred of 0: [1, 2] (in degree 2)
  List of neighbors of 1: [0] (degree 1)
    List of succ of 1: [3, 4] (out degree 2)
    List of pred of 1: [3, 4] (in degree 2)
  List of neighbors of 2: [0] (degree 1)
    List of succ of 2: [5, 6] (out degree 2)
    List of pred of 2: [5, 6] (in degree 2)
  List of neighbors of 3: [1] (degree 1)
    List of succ of 3: [] (out degree 0)
    List of pred of 3: [] (in degree 0)
  List of neighbors of 4: [1] (degree 1)
    List of succ of 4: [] (out degree 0)
    List of pred of 4: [] (in degree 0)
  List of neighbors of 5: [2] (degree 1)
    List of succ of 5: [] (out degree 0)
    List of pred of 5: [] (in degree 0)
  List of neighbors of 6: [2] (degree 1)
    List of succ of 6: [] (out degree 0)
    List of pred of 6: [] (in degree 0)
Creating empty graph with class <class '__main__.EdgesListGraph'>...
Adding vertex 0...
Adding vertex 1...
Adding vertex 2...
Adding vertex 3...
Adding vertex 4...
Adding vertex 5...
Adding vertex 6...
Adding edge (0, 1)...
Adding edge (0, 2)...
Adding edge (1, 3)...
Adding edge (1, 4)...
Adding edge (2, 5)...
Adding edge (2, 6)...
Graph:
{(0, 1), (1, 3), (2, 6), (1, 4), (2, 5), (0, 2)}
Number of vertexes: 7
Is the graph oriented? False
Number of edges: 12
```

```
List of vertexes: [0, 1, 2, 3, 4, 5, 6]
List of edges: {(0, 1), (1, 3), (2, 6), (3, 1), (5, 2), (1, 4), (2, 0), (2, 5), (6, 2), (1, 0)
  List of neighbors of 0: [1, 2] (degree 2)
    List of succ of 0: [1, 2] (out degree 2)
    List of pred of 0: [1, 2] (in degree 2)
  List of neighbors of 1: [0, 3, 4] (degree 3)
    List of succ of 1: [3, 4] (out degree 2)
    List of pred of 1: [3, 4] (in degree 2)
  List of neighbors of 2: [0, 5, 6] (degree 3)
    List of succ of 2: [5, 6] (out degree 2)
    List of pred of 2: [5, 6] (in degree 2)
  List of neighbors of 3: [1] (degree 1)
    List of succ of 3: [] (out degree 0)
    List of pred of 3: [] (in degree 0)
  List of neighbors of 4: [1] (degree 1)
    List of succ of 4: [] (out degree 0)
    List of pred of 4: [] (in degree 0)
  List of neighbors of 5: [2] (degree 1)
    List of succ of 5: [] (out degree 0)
    List of pred of 5: [] (in degree 0)
  List of neighbors of 6: [2] (degree 1)
    List of succ of 6: [] (out degree 0)
    List of pred of 6: [] (in degree 0)
```

3.3 Test numérique des complexités des différentes opérations

On rappelle qu'on devrait obtenir les résultats suivants, avec $n = |S|$ et $m = |A|$, que l'on va valider expérimentalement.

Opérations	Matrice d'adjacence	Listes d'adjacence	Liste d'arêtes
Création (vide)	temps et mémoire $O(n^2)$ si vide	temps et mémoire $O(n)$ si vide	temps et mémoire $O(1)$ si vide
Ajoute un sommet u	$O(n^2)$ (recopie)	$O(1)$	$O(1)$
Retire un sommet u	$O(n^2)$ (recopie)	$O(d(u))$ si orienté, $O(n+m)$ sinon	$O(n)$ (suppression des arêtes)
Ajoute un arc (u, v)	$O(1)$	$O(d(u))$ si orienté, $O(d(u)+d(v))$ sinon	$O(1)$ (si liste d'arêtes) ou $O(1)$ en amorti (si ensemble d'arêtes)
Retire un arc (u, v)	$O(1)$	$O(d(u))$ si orienté, $O(d(u)+d(v))$ sinon	$O(n)$ (si liste d'arêtes) ou $O(1)$ en amorti (si ensemble d'arêtes)

Opérations	Matrice d'adjacence	Listes d'adjacence	Liste d'arêtes
Liste des sommets	$O(n)$	$O(n)$	$O(n)$
Liste des arcs	$O(n^2)$ tout parcourir	$O(n)$ parcourir les n listes de tailles $d(u)$, et $\sum_u d(u) = n$	$O(1)$ (si liste d'arêtes) ou $O(n)$ (si ensemble d'arêtes)
Liste des voisins du nud u	$O(n)$	$O(d(u))$ ($O(1)$ si on ne crée pas de nouvelle liste)	$O(n)$
Degré du nud u	$O(n)$	$O(n)$	$O(n)$
Liste des voisins sortant du nud u	$O(n)$	$O(n)$	$O(n)$
Degré sortant du nud u	$O(n)$	$O(n)$	$O(n)$
Liste des voisins entrant du nud u	$O(n)$	$O(n)$	$O(n)$
Degré entrant du nud u	$O(n)$	$O(n)$	$O(n)$

```
In [62]: try:
             from tqdm import tqdm_notebook
         except ImportError:
             def tqdm_notebook(iterator, *args, **kwargs):
                 return iterator
```

```
In [64]: def random_vertex(n):
             return random.randint(0, n+1)

         def random_edge(n):
             return (random_vertex(n), random_vertex(n))
```

3.3.1 Un exemple

```
In [116]: probability = 0.5
          n = 1000
          graph = randomGraph(AdjMatrixGraph, n=n, probability=probability)
          %timeit graph.is_vertex(random_vertex(n))
          %timeit graph.add_vertex(n + 5)
          %timeit graph.add_edge(*random_edge(n))
          %timeit graph.remove_edge(*random_edge(n))
          %timeit graph.pred(random_vertex(n))
          %timeit graph.is_pred(*random_edge(n))
```

```
%timeit graph.succ(random_vertex(n))
%timeit graph.is_succ(*random_edge(n))
%timeit graph.neighbors(random_vertex(n))
%timeit graph.is_neighbor(*random_edge(n))
```

```
1.27 µs ś 26.7 ns per loop (mean ś std. dev. of 7 runs, 1000000 loops each)
267 ns ś 8.61 ns per loop (mean ś std. dev. of 7 runs, 1000000 loops each)
2.99 µs ś 408 ns per loop (mean ś std. dev. of 7 runs, 100000 loops each)
2.74 µs ś 45.8 ns per loop (mean ś std. dev. of 7 runs, 100000 loops each)
271 µs ś 6.93 µs per loop (mean ś std. dev. of 7 runs, 1000 loops each)
2.68 µs ś 91.6 ns per loop (mean ś std. dev. of 7 runs, 100000 loops each)
261 µs ś 7.3 µs per loop (mean ś std. dev. of 7 runs, 1000 loops each)
2.68 µs ś 74.1 ns per loop (mean ś std. dev. of 7 runs, 100000 loops each)
261 µs ś 9.08 µs per loop (mean ś std. dev. of 7 runs, 1000 loops each)
2.69 µs ś 91.6 ns per loop (mean ś std. dev. of 7 runs, 100000 loops each)
```

Sans voir l'évolution en fonction de n, difficile de conclure quoi que ce soit de ces premières expériences…

3.3.2 Tests pour différentes tailles de graphes

On va stocker les temps de calculs dans une petite structure de la forme suivante, qui permettra d'afficher directement des courbes (le traitement est fait plus bas).

```
In [80]: times = {
             # clé sur n
             "100": {
                 # clé sur p
                 r"|A| \simeq |S|": {
                     "AdjMatrixGraph": {
                         "operation1": 0.12,
                         # ...,
                         "operationN": 0.12,
                     },
                     "AdjListsGraph": {
                         "operation1": 0.12,
                         # ...,
                         "operationN": 0.12,
                     },
                     "EdgesListGraph": {
                         "operation1": 0.12,
                         # ...,
                         "operationN": 0.12,
                     },
                 },
             },
         }
```

```
In [117]: import timeit

In [118]: times = {}

          for n in tqdm_notebook([100, 200, 400, 800, 1000, 1500, 2000, 2500, 3000, 3500, 4000
              number = 20 if n <= 200 else 5  # nb of repetitions of each operations
              # print(f"\n\n For graphs with {n} vertexes:")
              times[n] = {}
              for probability, pname in tqdm_notebook([
                  (1.0/n, r"|A| \simeq |S|"),  # p = 1/n => |A| ~= |S|
                  (1.0/np.sqrt(n), r"|A| \simeq |S|^{3/2}"),  # p = 1/sqrt(n) => |A| ~= |S|^(3
                  (0.1, r"|A| \simeq 0.1 |S|^2"),  # p = 0.1 => |A| ~= 0.1 |S|^2
              ], desc="proba"):
                  times[n][pname] = {}
                  # print(f"\n  and link probability of {probability}:")
                  for GraphClass in [AdjMatrixGraph, AdjListsGraph, EdgesListGraph]:
                      # print(f"\n    for class {GraphClass}...")
                      graph = randomGraph(GraphClass, n=n, probability=probability)
                      the_times = {}
                      the_times["randomGraph"] = timeit.timeit(
                          "randomGraph(GraphClass, n=n, probability=probability)",
                          globals=globals(), number=number,
                      )
                      # print("Time to create a new graph:", the_times["randomGraph"])
                      the_times["is_vertex"] = timeit.timeit(
                          "graph.is_vertex(random_vertex(n))",
                          globals=globals(), number=number,
                      )
                      # print("Time to test presence of a vertex:", the_times["is_vertex"])
                      the_times["add_vertex"] = timeit.timeit(
                          "graph.add_vertex(n + 2)",
                          globals=globals(), number=number,
                      )
                      # print("Time to add the next vertex n + 2:", the_times["add_vertex"])
                      the_times["pred"] = timeit.timeit(
                          "graph.pred(random_vertex(n))",
                          globals=globals(), number=number,
                      )
                      # print("Time to compute pred:", the_times["pred"])
                      the_times["is_pred"] = timeit.timeit(
                          "graph.is_pred(*random_edge(n))",
                          globals=globals(), number=number,
                      )
                      # print("Time to test pred:", the_times["is_pred"])
                      the_times["succ"] = timeit.timeit(
                          "graph.succ(random_vertex(n))",
                          globals=globals(), number=number,
                      )
```

```python
                # print("Time to compute succ:", the_times["succ"])
                the_times["is_succ"] = timeit.timeit(
                    "graph.is_succ(*random_edge(n))",
                    globals=globals(), number=number,
                )
                # print("Time to test succ:", the_times["is_succ"])
                the_times["neighbors"] = timeit.timeit(
                    "graph.neighbors(random_vertex(n))",
                    globals=globals(), number=number,
                )
                # print("Time to compute neighbors:", the_times["neighbors"])
                the_times["is_neighbor"] = timeit.timeit(
                    "graph.is_neighbor(*random_edge(n))",
                    globals=globals(), number=number,
                )
                # print("Time to test neighbors:", the_times["is_neighbor"])
                the_times["add_edge"] = timeit.timeit(
                    "graph.add_edge(*random_edge(n))",
                    globals=globals(), number=number,
                )
                # print("Time to add an edge:", the_times["add_edge"])
                times[n][pname][str(GraphClass)] = the_times
```

```
HBox(children=(IntProgress(value=0, description='n', max=13, style=ProgressStyle(description_wi

HBox(children=(IntProgress(value=0, description='proba', max=3, style=ProgressStyle(descriptio

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```

```
HBox(children=(IntProgress(value=0, description='proba', max=3, style=ProgressStyle(description
```

3.3.3 Afficher ces mesures de temps de complexités

```
In [103]: import matplotlib.pyplot as plt
          import seaborn as sns
          sns.set(context="notebook", style="whitegrid", palette="hls", font="sans-serif", font
          import matplotlib as mpl
          mpl.rcParams['figure.figsize'] = (10, 7)
          mpl.rcParams['figure.dpi'] = 120
```

Il faut écrire une fonction qui va extraire les données de ce `times`, et les afficher.

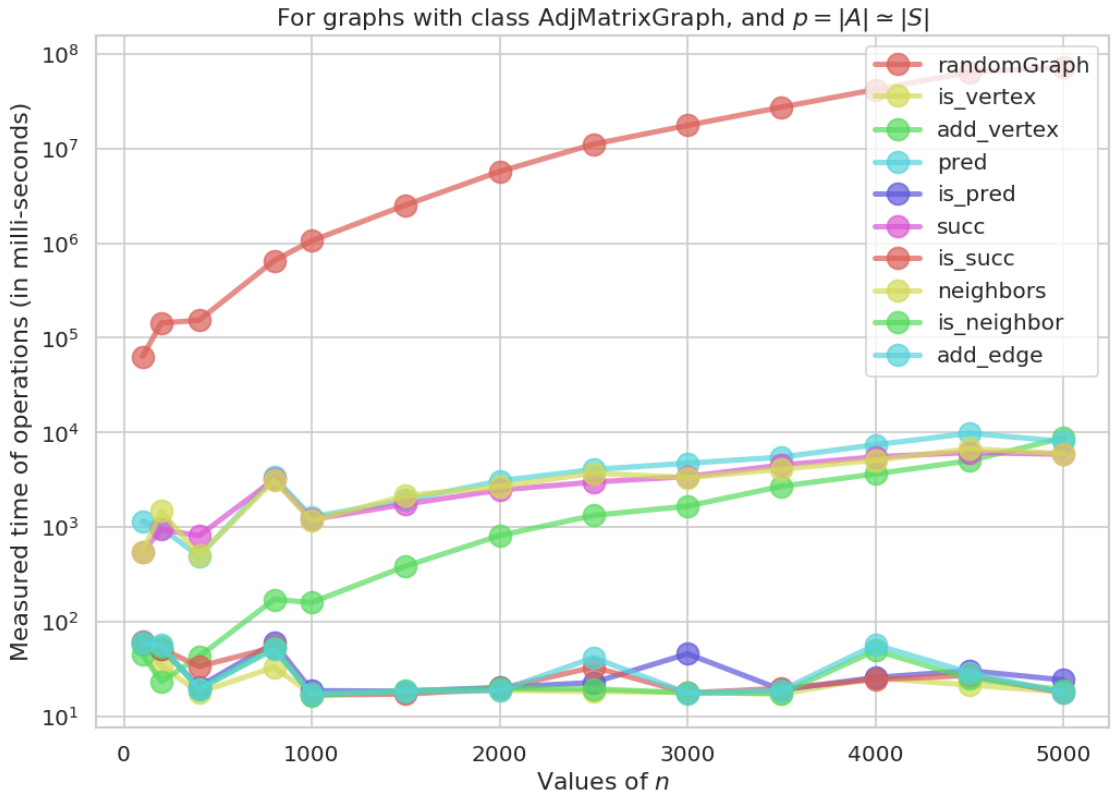
- J'ai choisi d'afficher une courbe différente pour chaque valeur de p, et de structure de données,
- Et sur chaque courbe, il y aura n le nombre de sommets du graphe en abscisse, le temps (en milli secondes) en ordonnées, et des courbes pour chaque opérations.

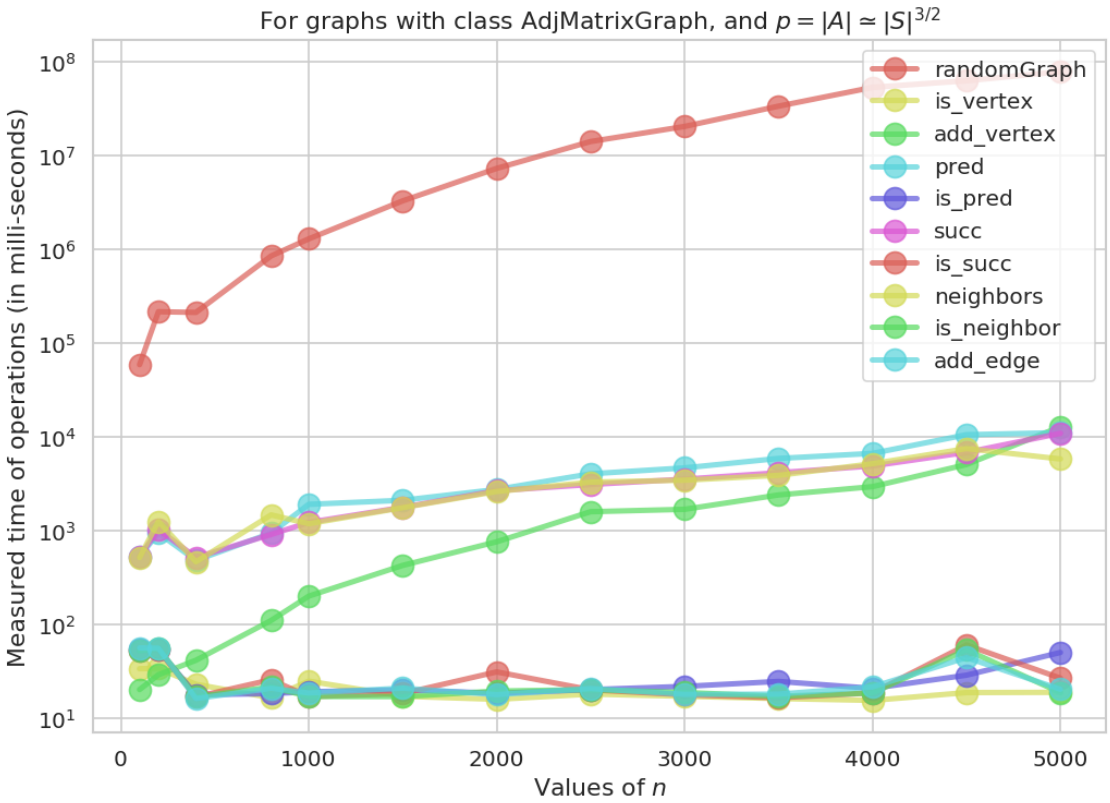
```
In [193]: def plotComplexitiesOfOperations(times):
              values_n = list(times.keys())
              values_p = list(times[values_n[0]].keys())
              values_class = list(times[values_n[0]][values_p[0]].keys())
              values_opname = list(times[values_n[0]][values_p[0]][values_class[0]].keys())
              for class_name in values_class:
                  for pname in values_p:
                      data = {
                          n: times[n][pname][class_name]
                          for n in values_n
                      }
                      fig = plt.figure()
                      for opname in values_opname:
                          plt.semilogy(values_n, [ 1e6 * data[n][opname] for n in values_n ],
                                       label=opname,
                                       marker='o', lw=3, ms=12, alpha=0.7,
                                      )
                      plt.xlabel("Values of $n$")
```

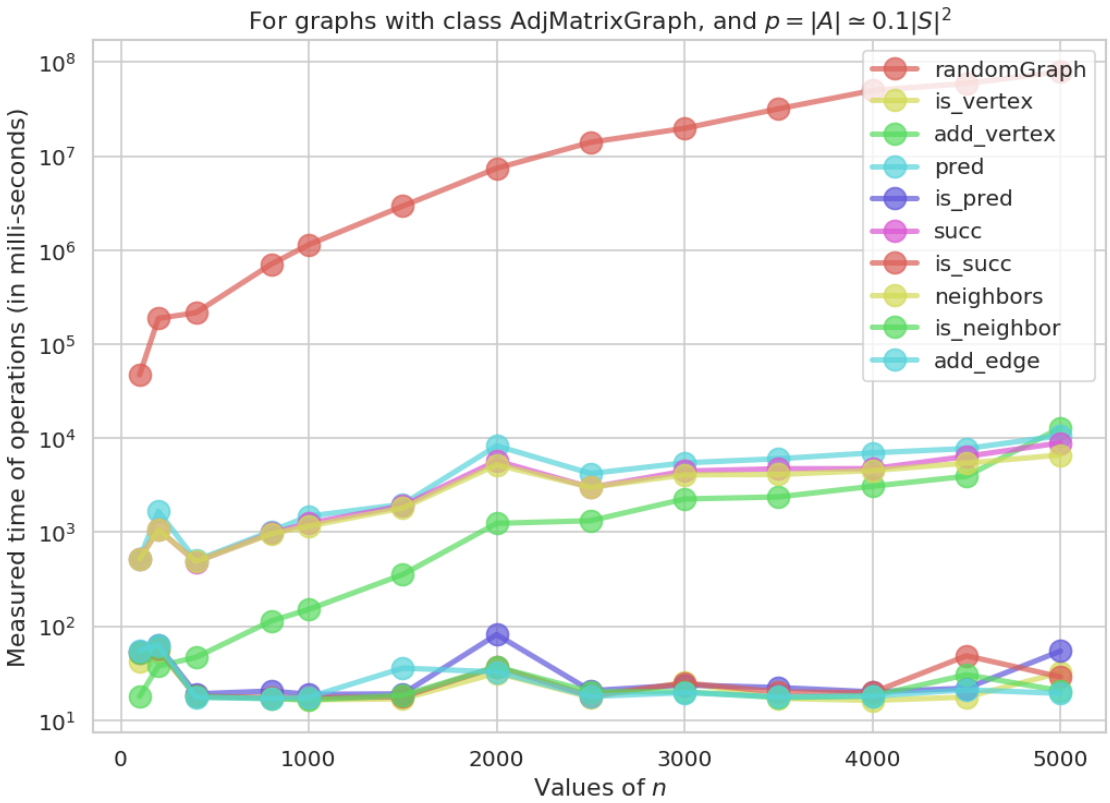
```
            plt.ylabel("Measured time of operations (in milli-seconds)")
            name = class_name.replace("<class '__main__.", "").replace("'>", "")
            plt.title(f"For graphs with class {name}, and $p={pname}$")
            plt.legend()
            plt.show()
    return fig
```

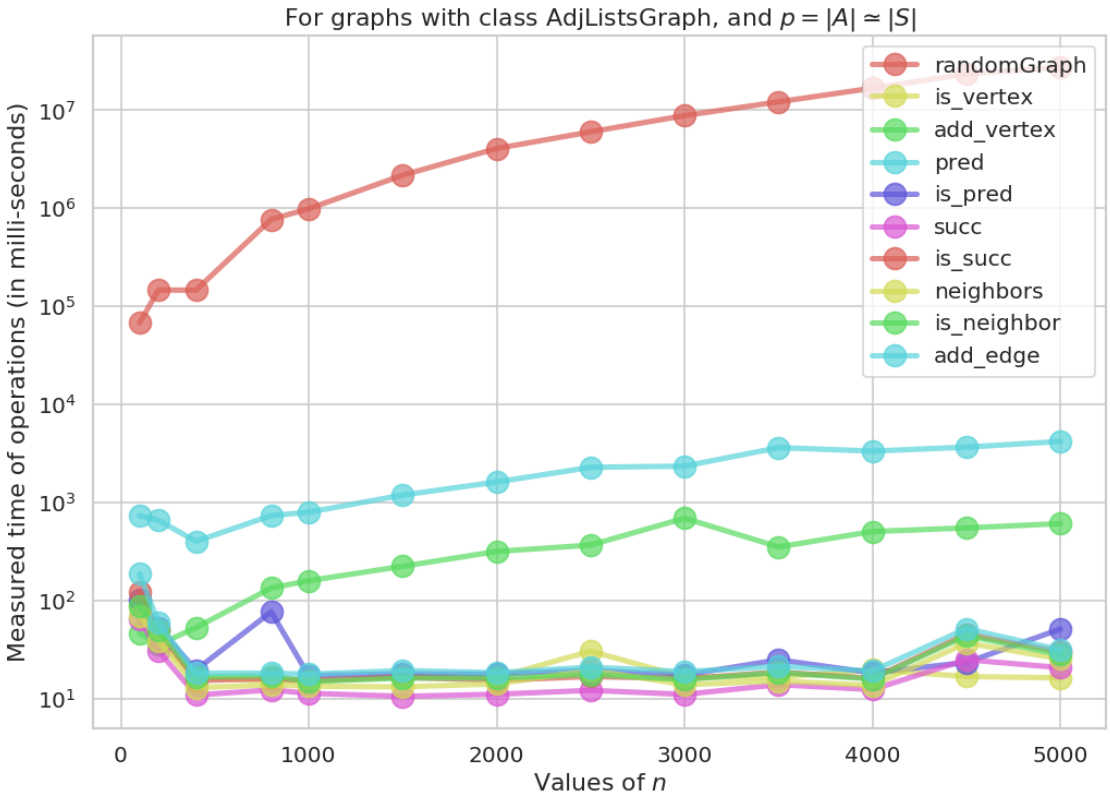
On vérifie cela :

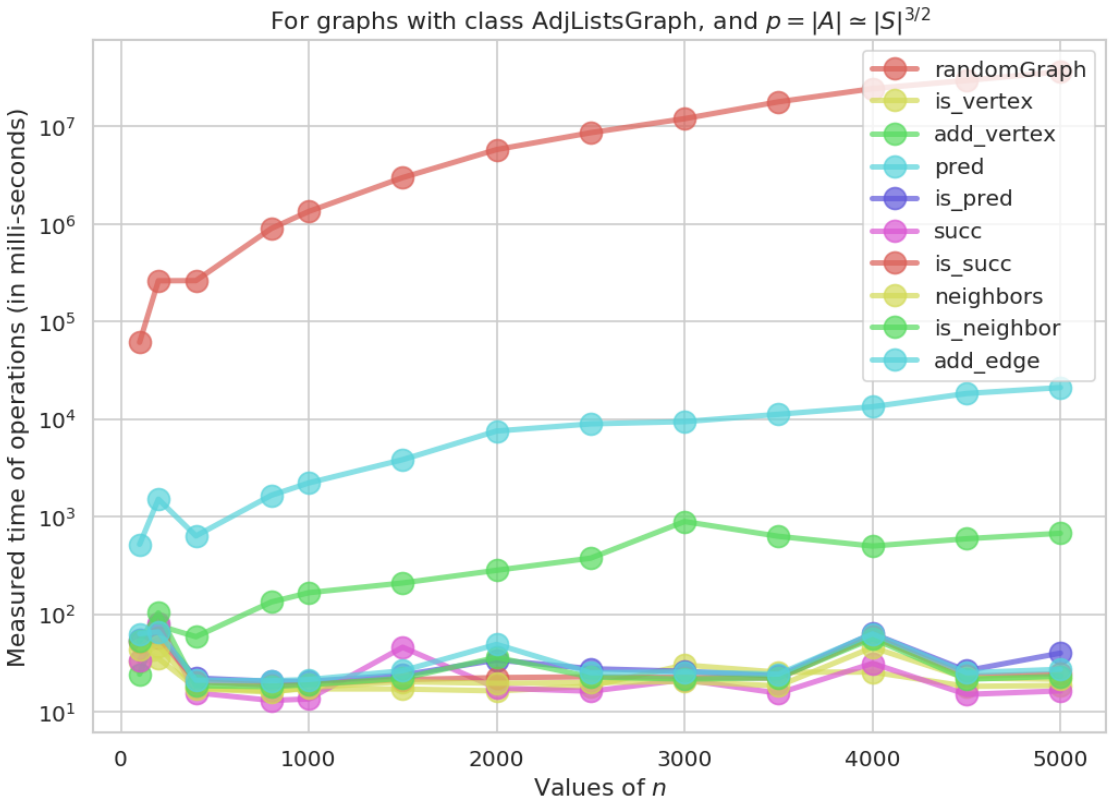
```
In [194]: _ = plotComplexitiesOfOperations(times)
```

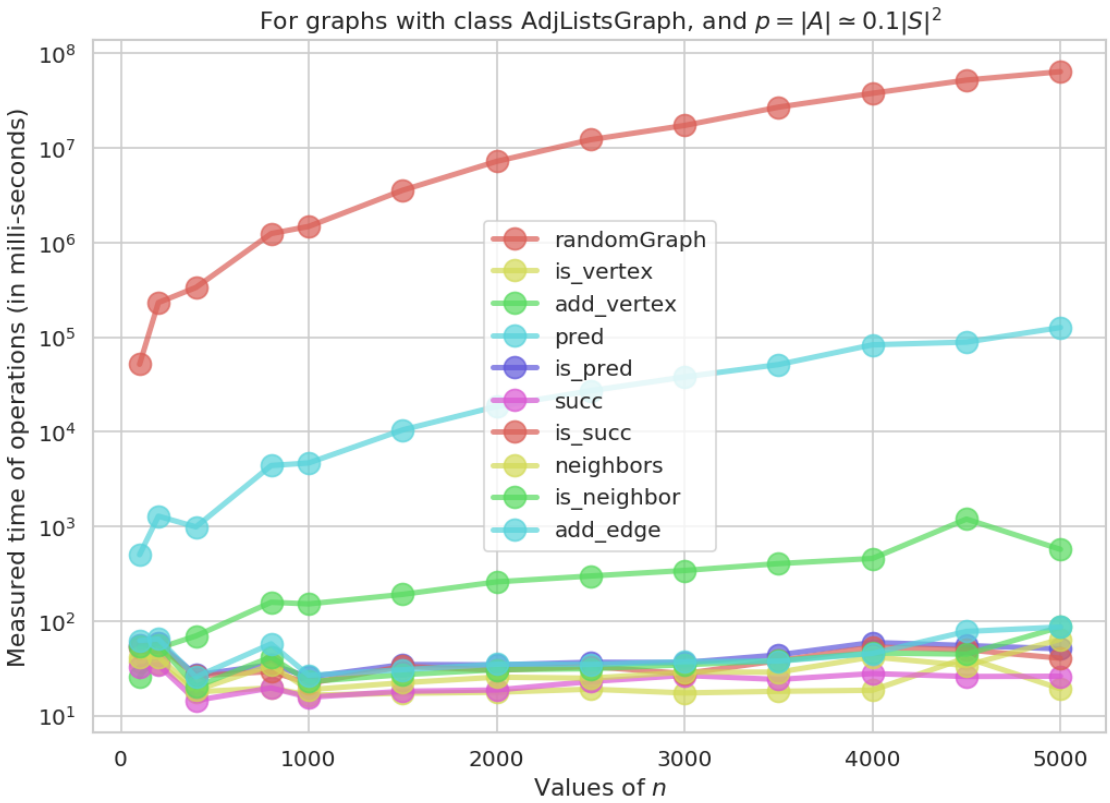


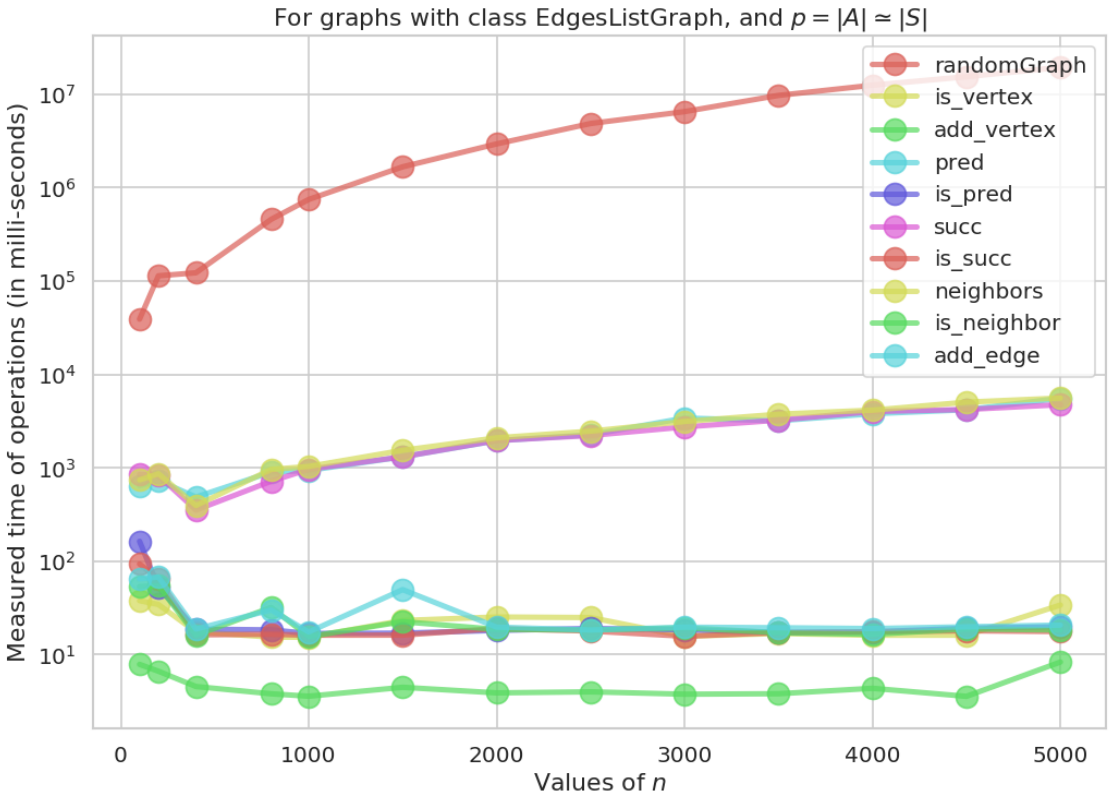

For graphs with class AdjMatrixGraph, and $p = |A| \simeq |S|^{3/2}$
Measured time of operations (in milli-seconds)
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0
1000
2000
3000
4000
5000
Values of n
randomGraph
is_vertex
add_vertex
pred
is_pred
succ
is_succ
neighbors
is_neighbor
add_edge

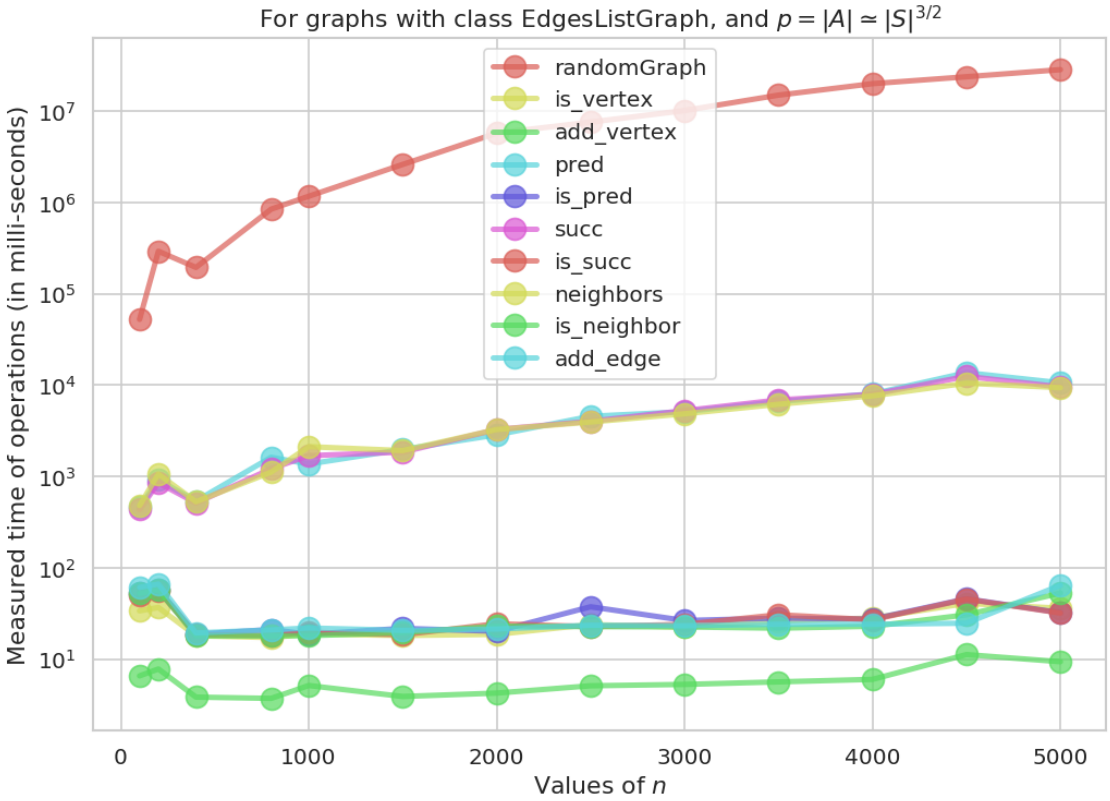

For graphs with class AdjMatrixGraph, and $p = |A| \simeq 0.1|S|^2$
Measured time of operations (in milli-seconds)
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0
1000
2000
3000
4000
5000
Values of n
randomGraph
is_vertex
add_vertex
pred
is_pred
succ
is_succ
neighbors
is_neighbor
add_edge

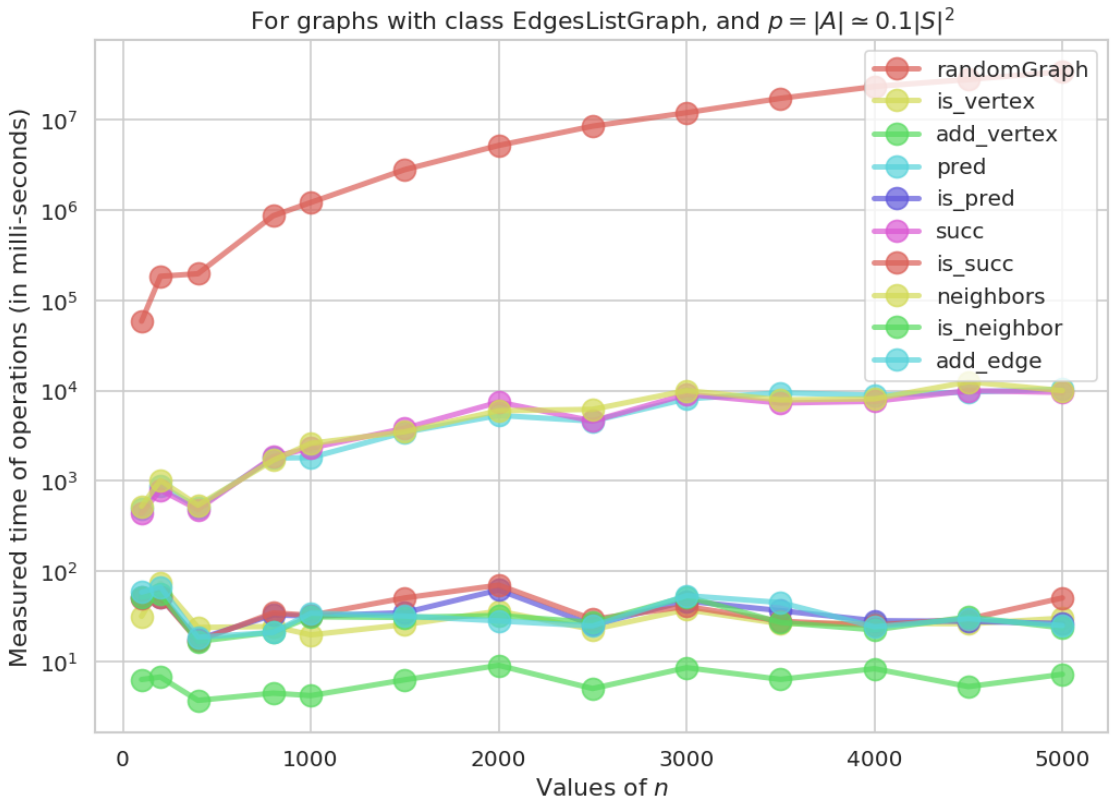

For graphs with class AdjListsGraph, and $p = |A| \simeq |S|$
Measured time of operations (in milli-seconds)
10^1
10^2
10^3
10^4
10^5
10^6
10^7
0
1000
2000
3000
4000
5000
Values of n
randomGraph
is_vertex
add_vertex
pred
is_pred
succ
is_succ
neighbors
is_neighbor
add_edge


For graphs with class AdjListsGraph, and $p = |A| \simeq |S|^{3/2}$
Measured time of operations (in milli-seconds)
10^1
10^2
10^3
10^4
10^5
10^6
10^7
0
1000
2000
3000
4000
5000
Values of n
randomGraph
is_vertex
add_vertex
pred
is_pred
succ
is_succ
neighbors
is_neighbor
add_edge


For graphs with class AdjListsGraph, and $p = |A| \simeq 0.1|S|^2$
Measured time of operations (in milli-seconds)
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
randomGraph
is_vertex
add_vertex
pred
is_pred
succ
is_succ
neighbors
is_neighbor
add_edge
0
1000
2000
3000
4000
5000
Values of n


For graphs with class EdgesListGraph, and $p = |A| \simeq |S|$
Measured time of operations (in milli-seconds)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0
1000
2000
3000
4000
5000
Values of n
randomGraph
is_vertex
add_vertex
pred
is_pred
succ
is_succ
neighbors
is_neighbor
add_edge


For graphs with class EdgesListGraph, and $p = |A| \simeq |S|^{3/2}$
randomGraph
is_vertex
add_vertex
pred
is_pred
succ
is_succ
neighbors
is_neighbor
add_edge
Measured time of operations (in milli-seconds)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0
1000
2000
3000
4000
5000
Values of n



Il faut écrire une fonction qui va extraire les données de ce `times`, et les afficher.

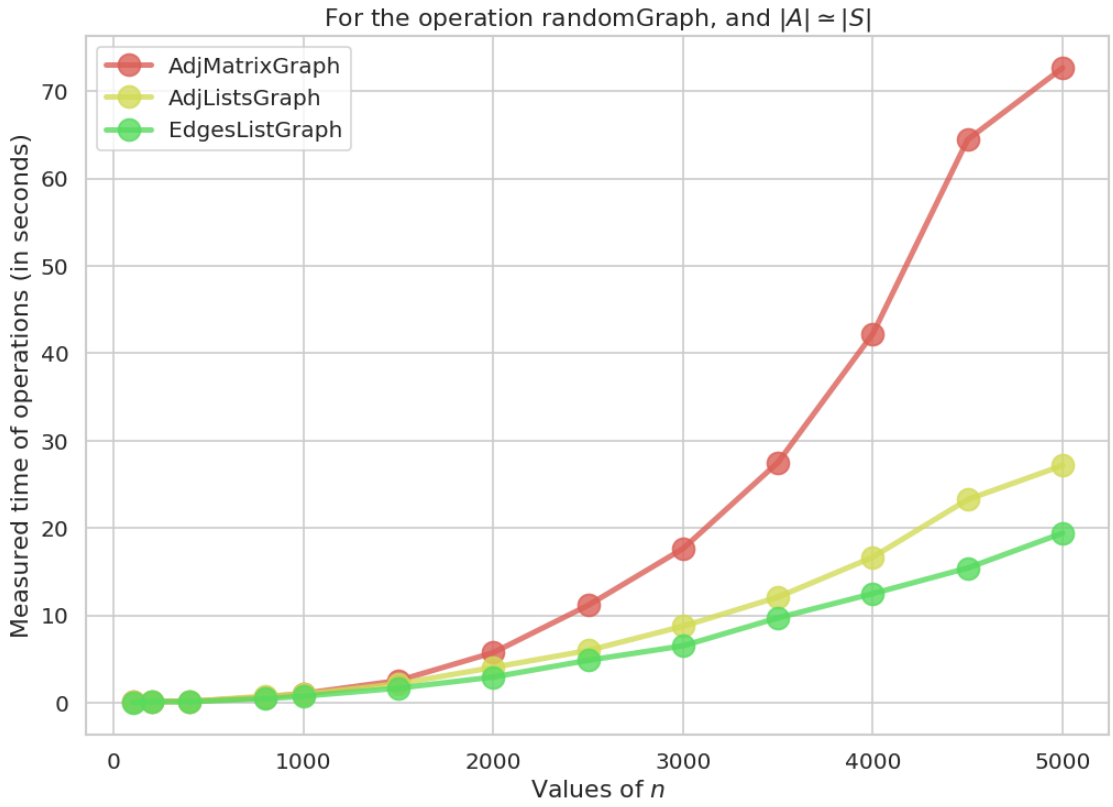
- J'ai choisi d'afficher une courbe différente pour chaque valeur de p, et de structure de données,
- Et sur chaque courbe, il y aura n le nombre de sommets du graphe en abscisse, le temps (en milli secondes) en ordonnées, et des courbes pour chaque opérations.

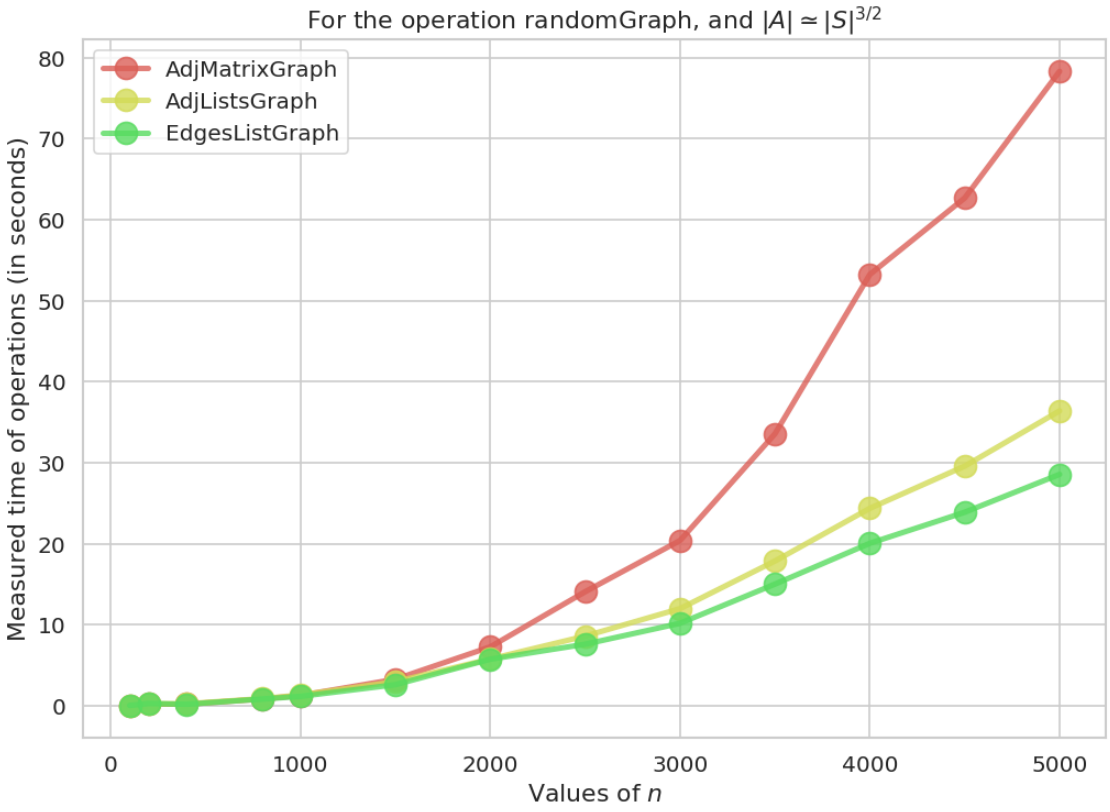
```
In [195]: def plotComplexitiesOfOperations2(times):
              values_n = list(times.keys())
              values_p = list(times[values_n[0]].keys())
              values_class = list(times[values_n[0]][values_p[0]].keys())
              values_opname = list(times[values_n[0]][values_p[0]][values_class[0]].keys())
              for opname in values_opname:
                      for pname in values_p:
                          fig = plt.figure()
                          for class_name in values_class:
                              name = class_name.replace("<class '__main__.", "").replace("'>",
                              plt.plot(values_n, [ times[n][pname][class_name][opname] for n in
                                       label=name, marker='o',
                                       lw=3, ms=12, alpha=0.8)
                          plt.xlabel("Values of $n$")
                          plt.ylabel("Measured time of operations (in seconds)")
                          plt.title(f"For the operation {opname}, and ${pname}$")
```

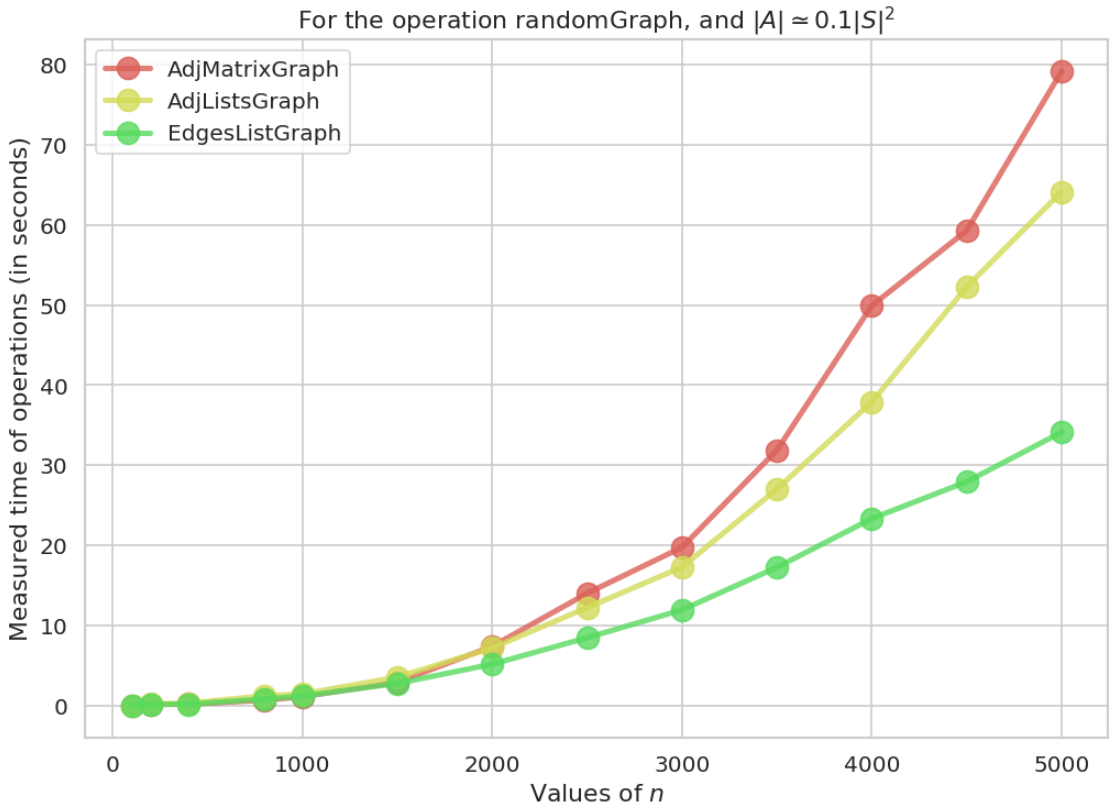
```
                plt.legend()
                plt.show()
        return fig
```

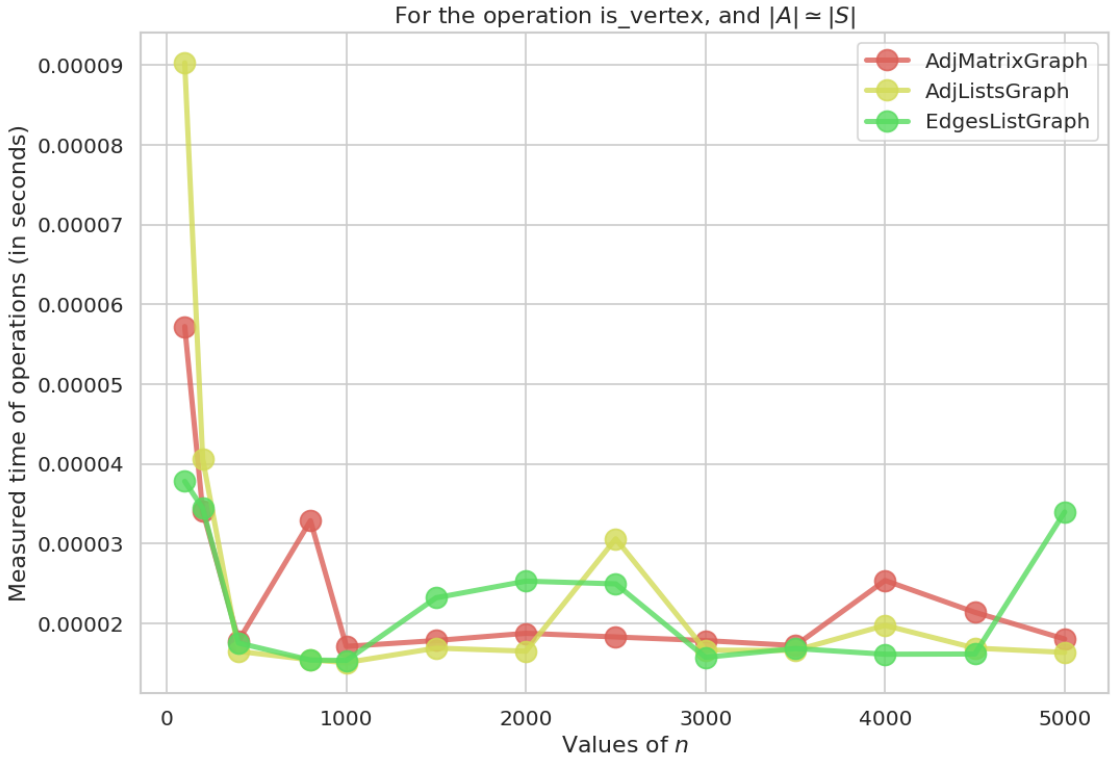
On vérifie cela :

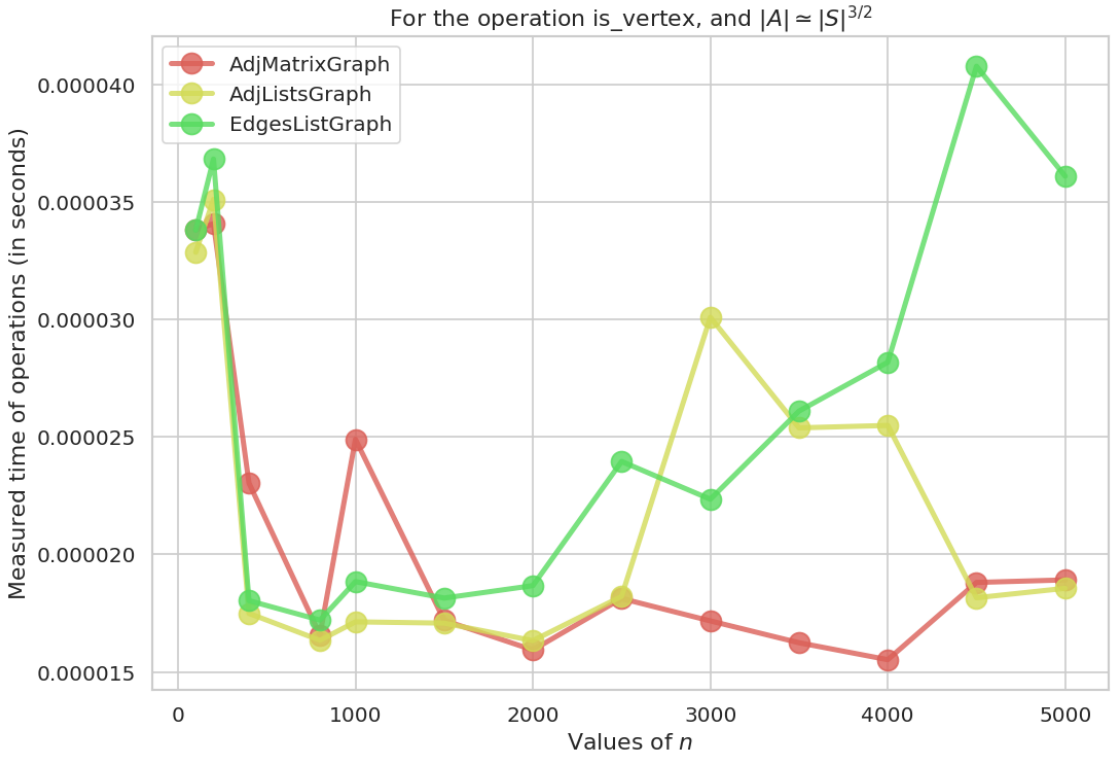
```
In [196]: _ = plotComplexitiesOfOperations2(times)
```

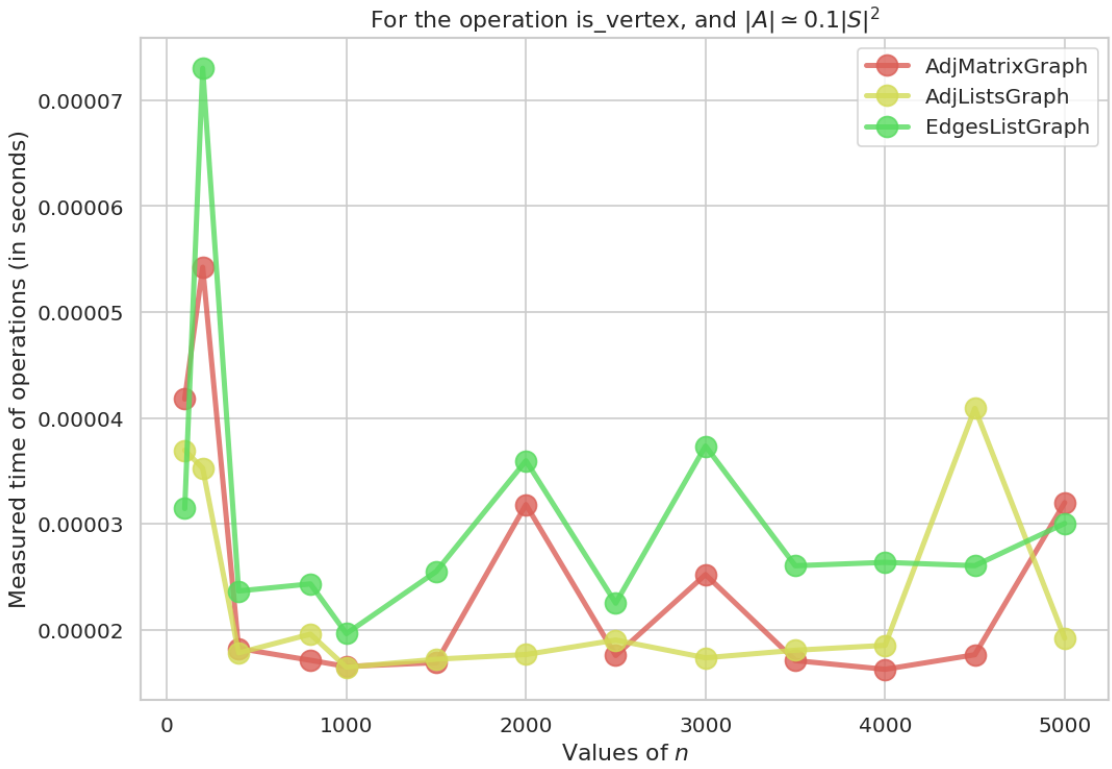


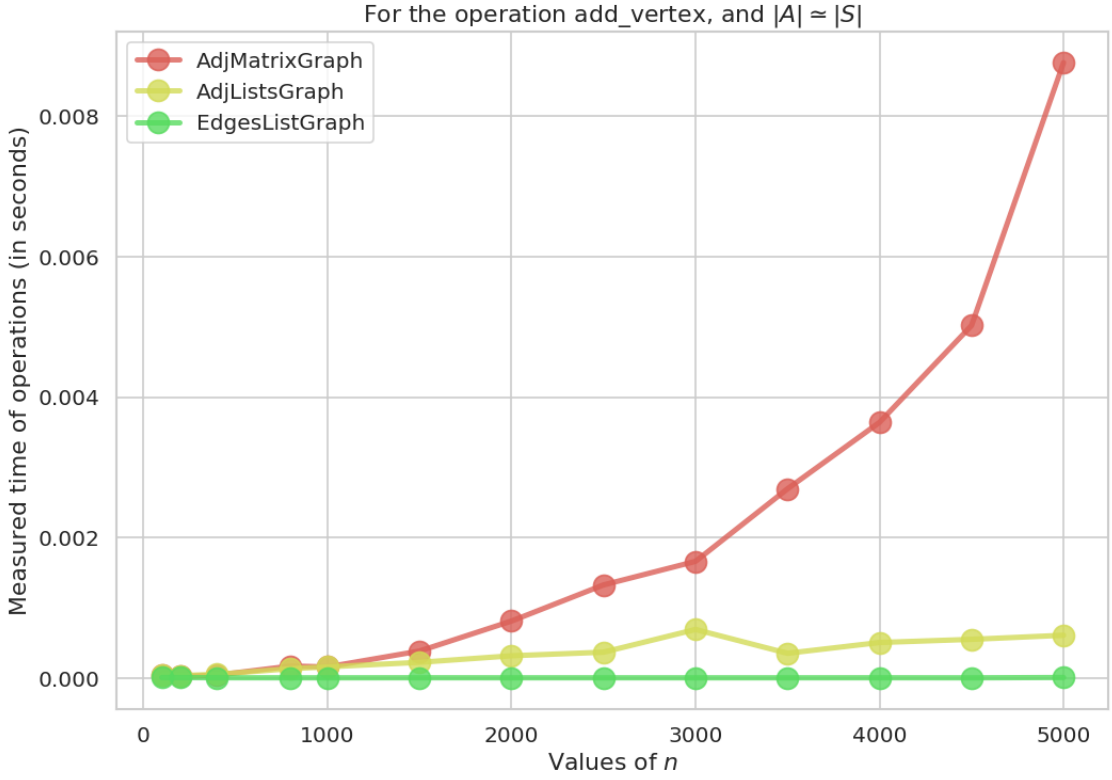

For the operation randomGraph, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
Values of n
0
10
20
30
40
50
60
70
80
0
1000
2000
3000
4000
5000

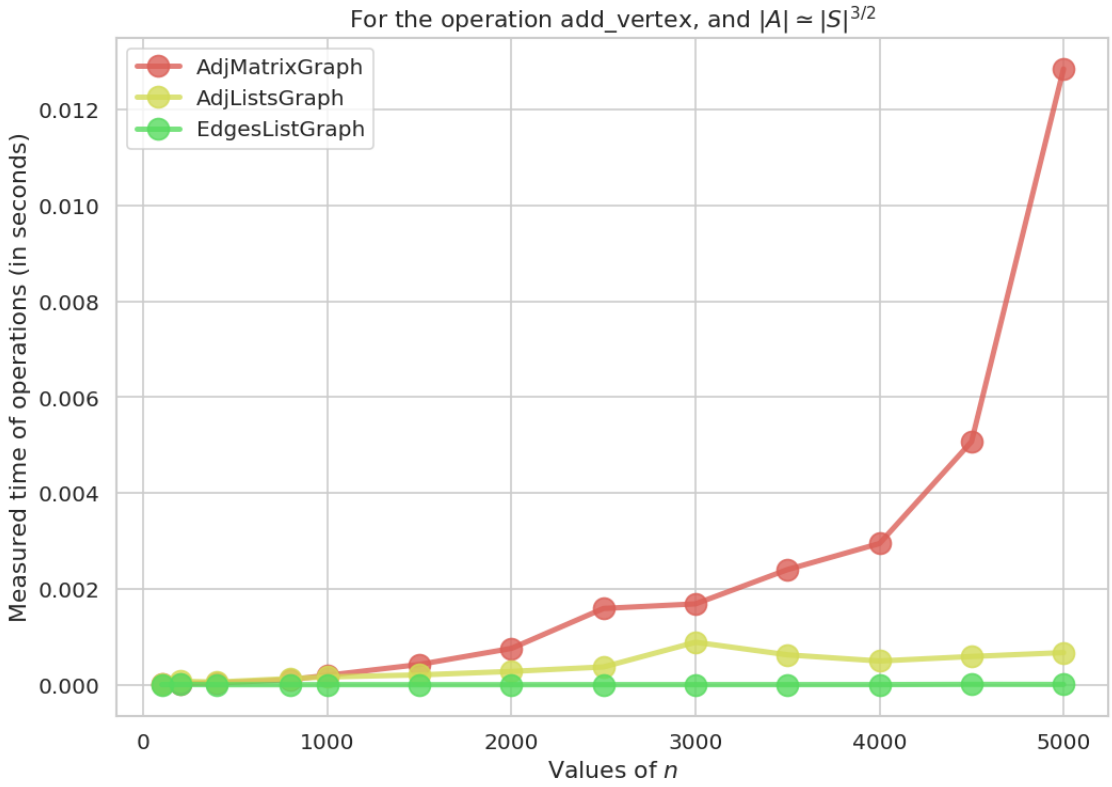

For the operation randomGraph, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
Values of n
0
10
20
30
40
50
60
70
80
0
1000
2000
3000
4000
5000

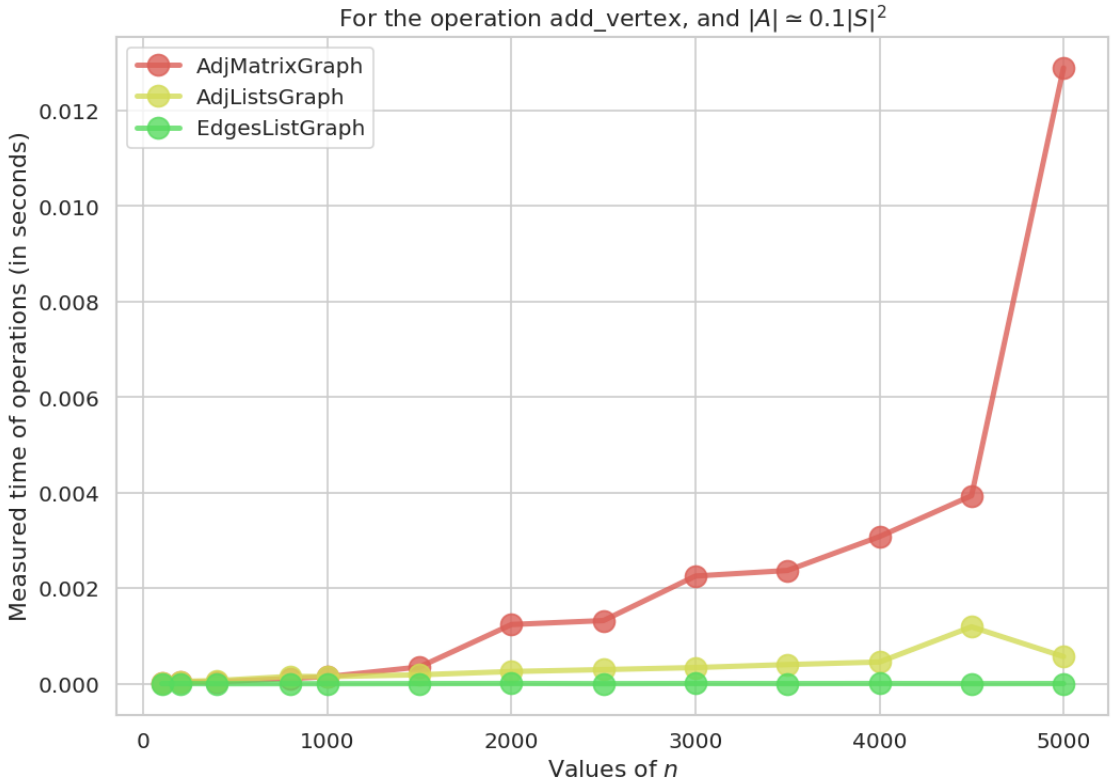

For the operation is_vertex, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
Values of n
0.00002
0.00003
0.00004
0.00005
0.00006
0.00007
0.00008
0.00009
0
1000
2000
3000
4000
5000

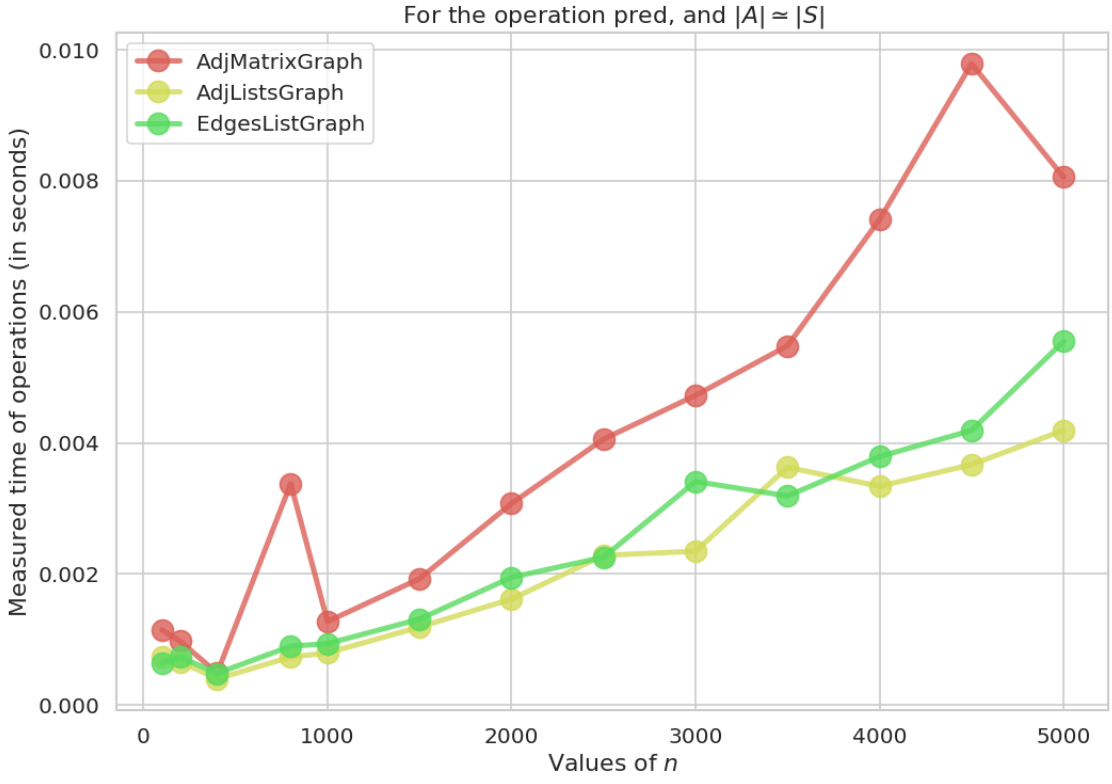

For the operation is_vertex, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.000040
0.000035
0.000030
0.000025
0.000020
0.000015
0
1000
2000
3000
4000
5000
Values of n

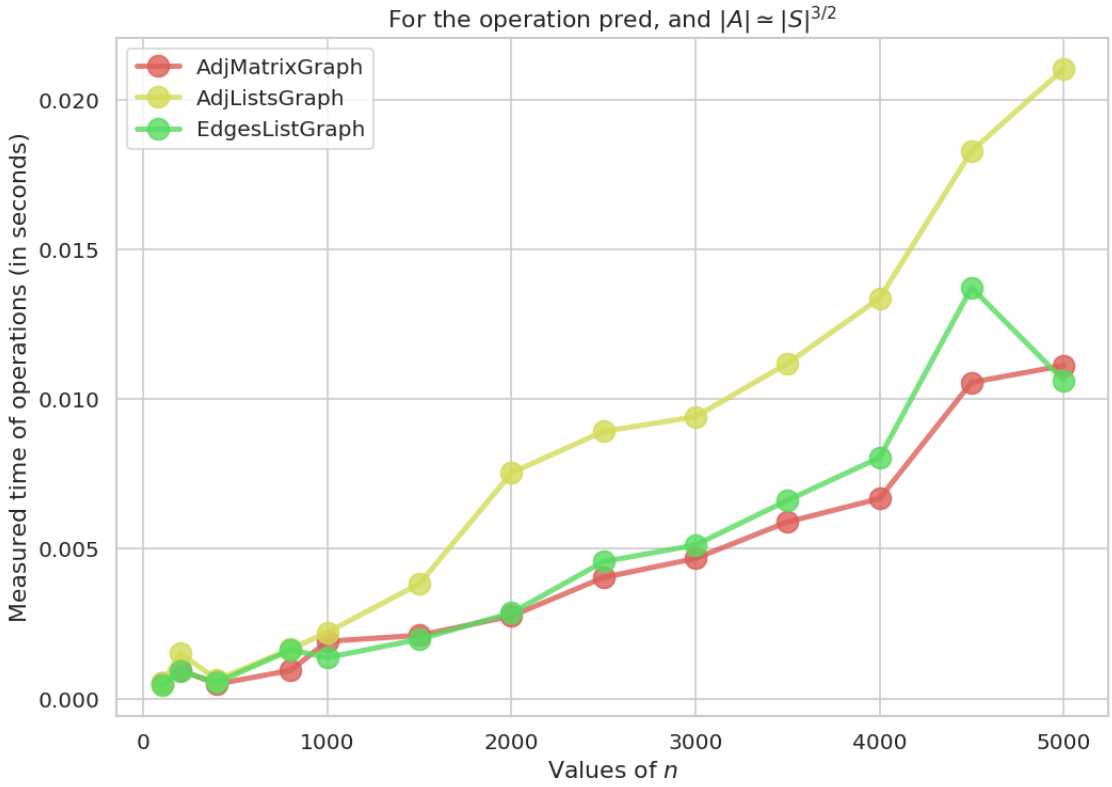

For the operation is_vertex, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00007
0.00006
0.00005
0.00004
0.00003
0.00002
0
1000
2000
3000
4000
5000
Values of n

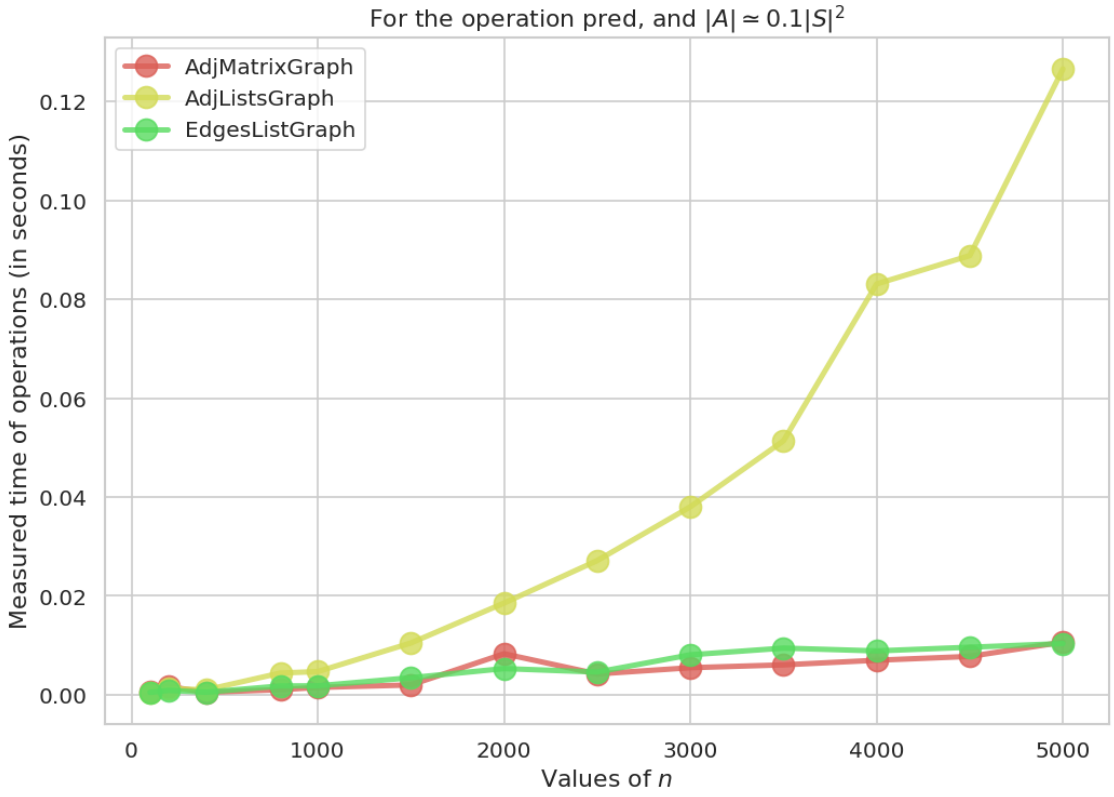

For the operation add_vertex, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.008
0.006
0.004
0.002
0.000
0
1000
2000
3000
4000
5000
Values of n


For the operation add_vertex, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0
1000
2000
3000
4000
5000
Values of n

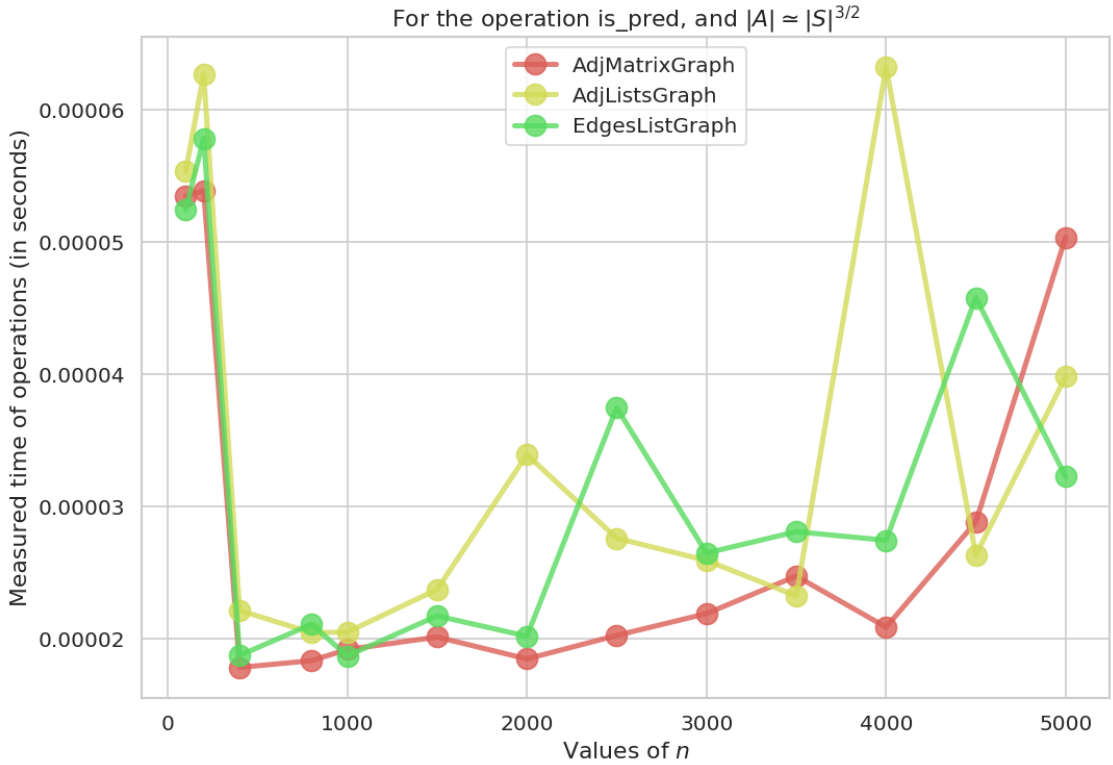

For the operation add_vertex, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.000
0.002
0.004
0.006
0.008
0.010
0.012
0
1000
2000
3000
4000
5000
Values of n

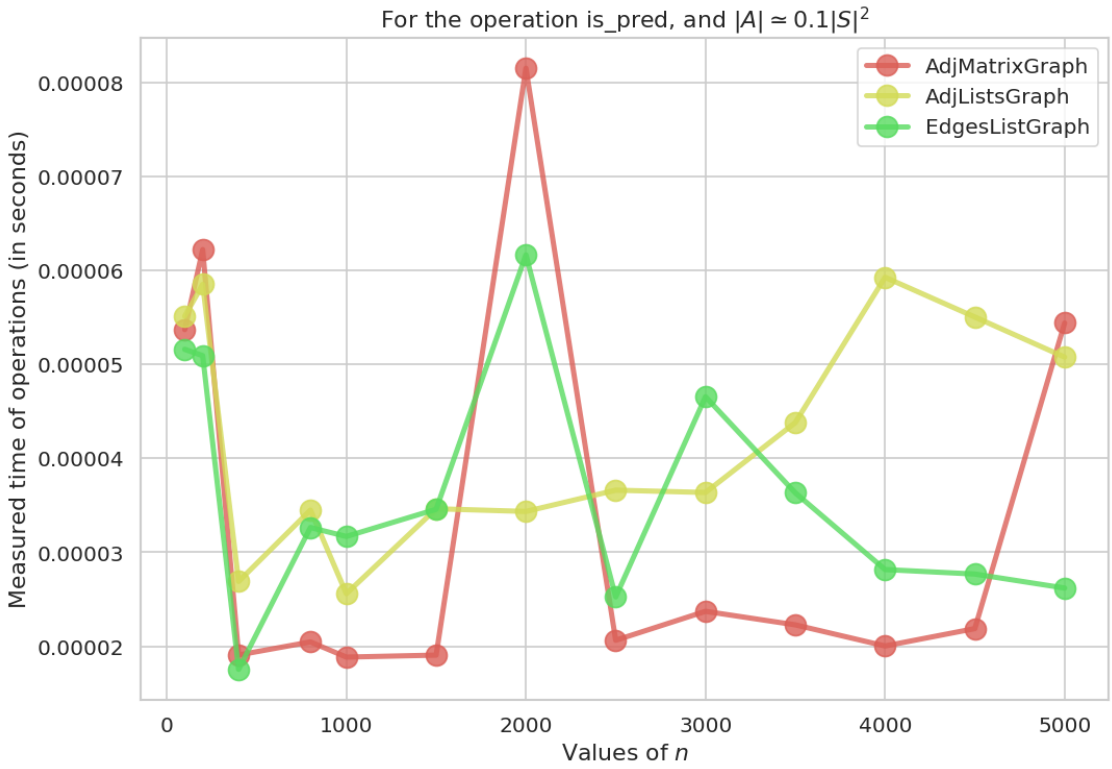

For the operation pred, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.000
0.002
0.004
0.006
0.008
0.010
0
1000
2000
3000
4000
5000
Values of n

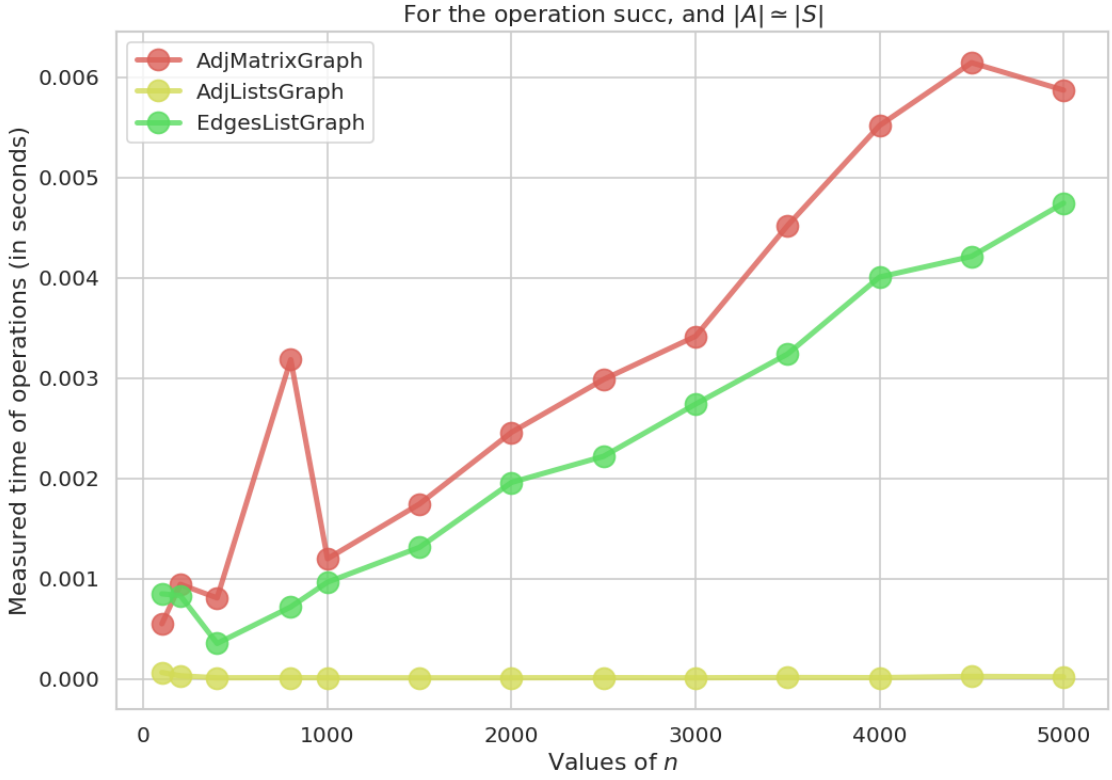

For the operation pred, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.000
0.005
0.010
0.015
0.020
0
1000
2000
3000
4000
5000
Values of n

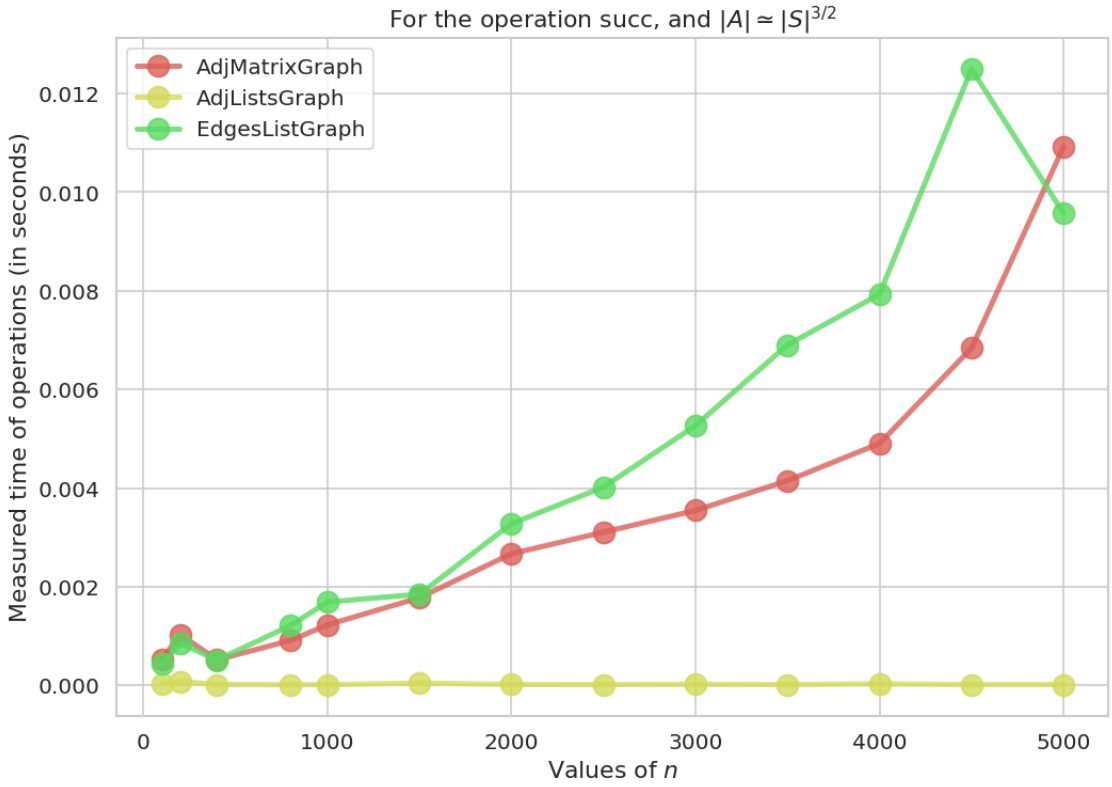

For the operation pred, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0
1000
2000
3000
4000
5000
Values of n


For the operation is_pred, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00002
0.00004
0.00006
0.00008
0.00010
0.00012
0.00014
0.00016
0
1000
2000
3000
4000
5000
Values of n


For the operation is_pred, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00006
0.00005
0.00004
0.00003
0.00002
0
1000
2000
3000
4000
5000
Values of n

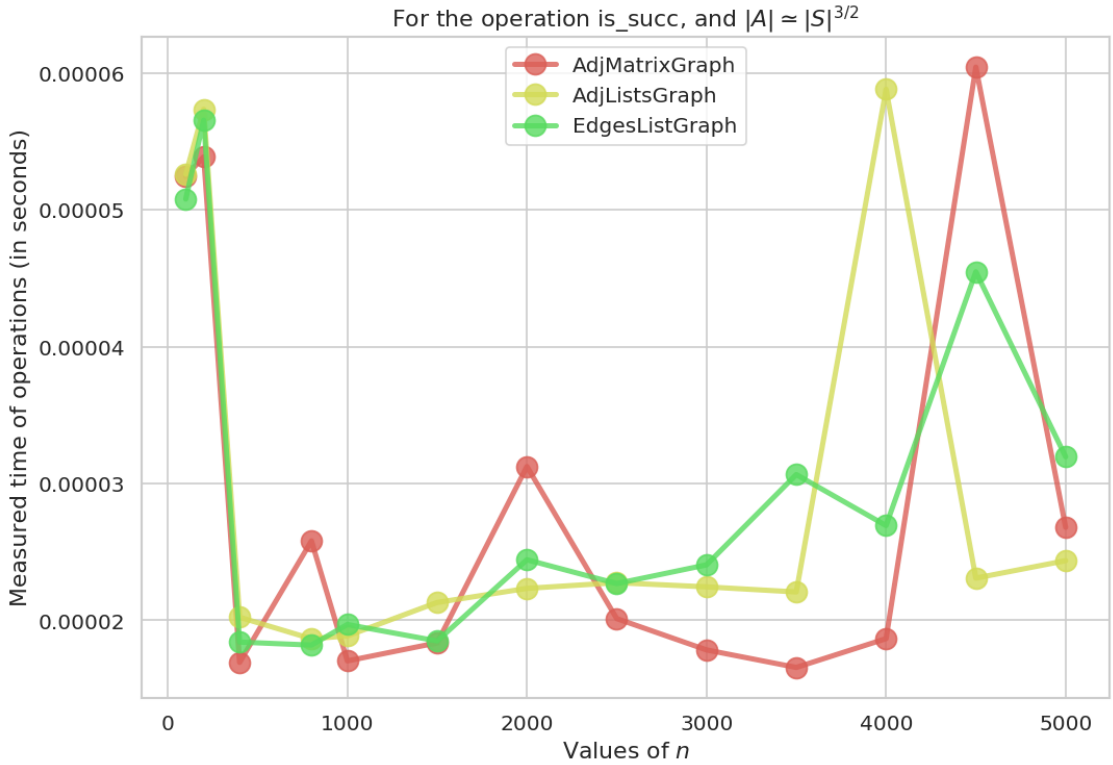

For the operation is_pred, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00008
0.00007
0.00006
0.00005
0.00004
0.00003
0.00002
0
1000
2000
3000
4000
5000
Values of n

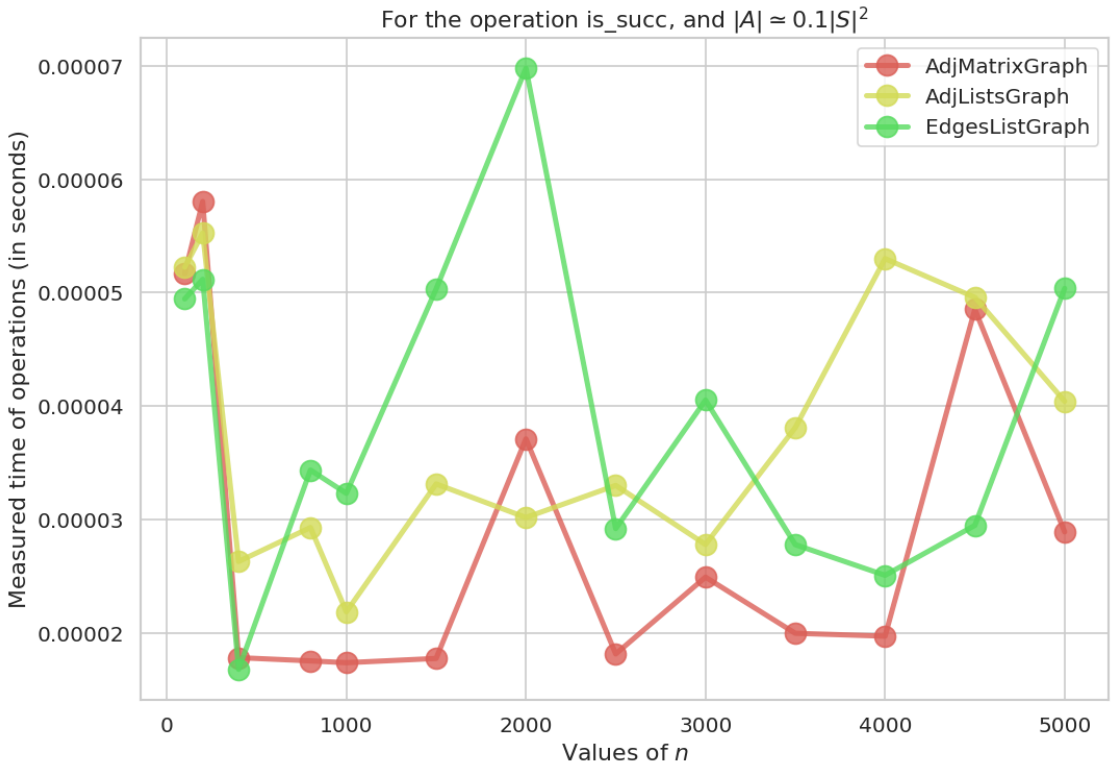

For the operation succ, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0
1000
2000
3000
4000
5000
Values of n

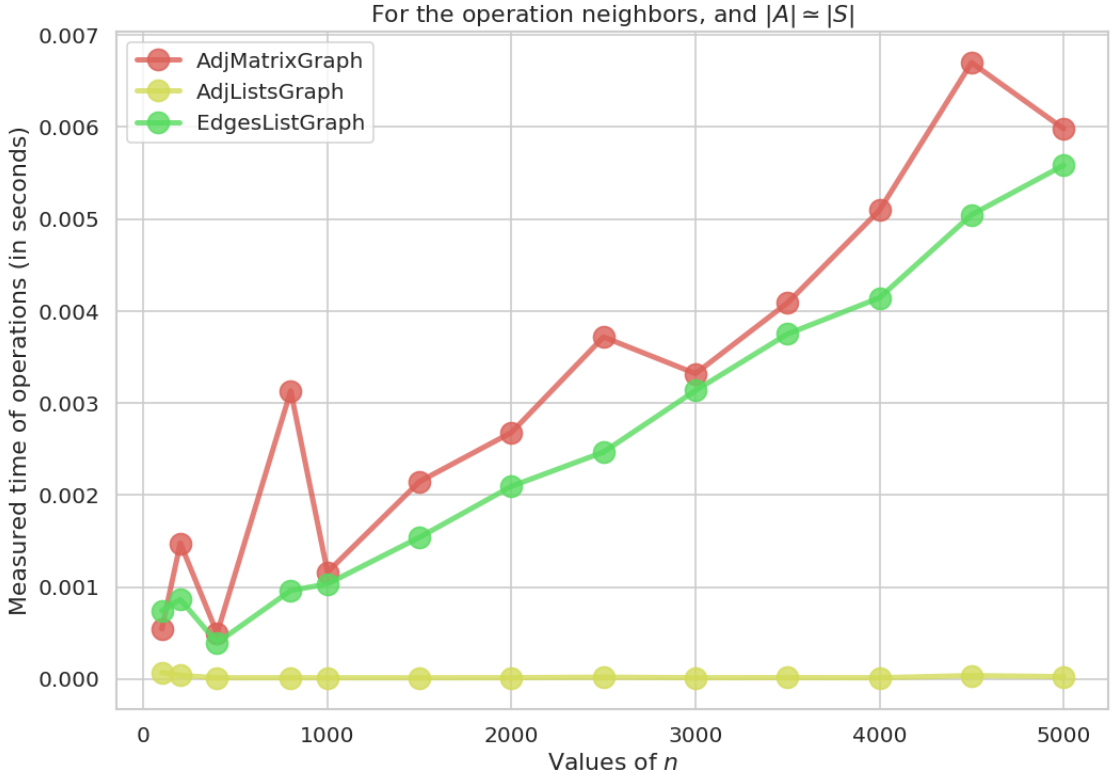

For the operation succ, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.000
0.002
0.004
0.006
0.008
0.010
0.012
0
1000
2000
3000
4000
5000
Values of n

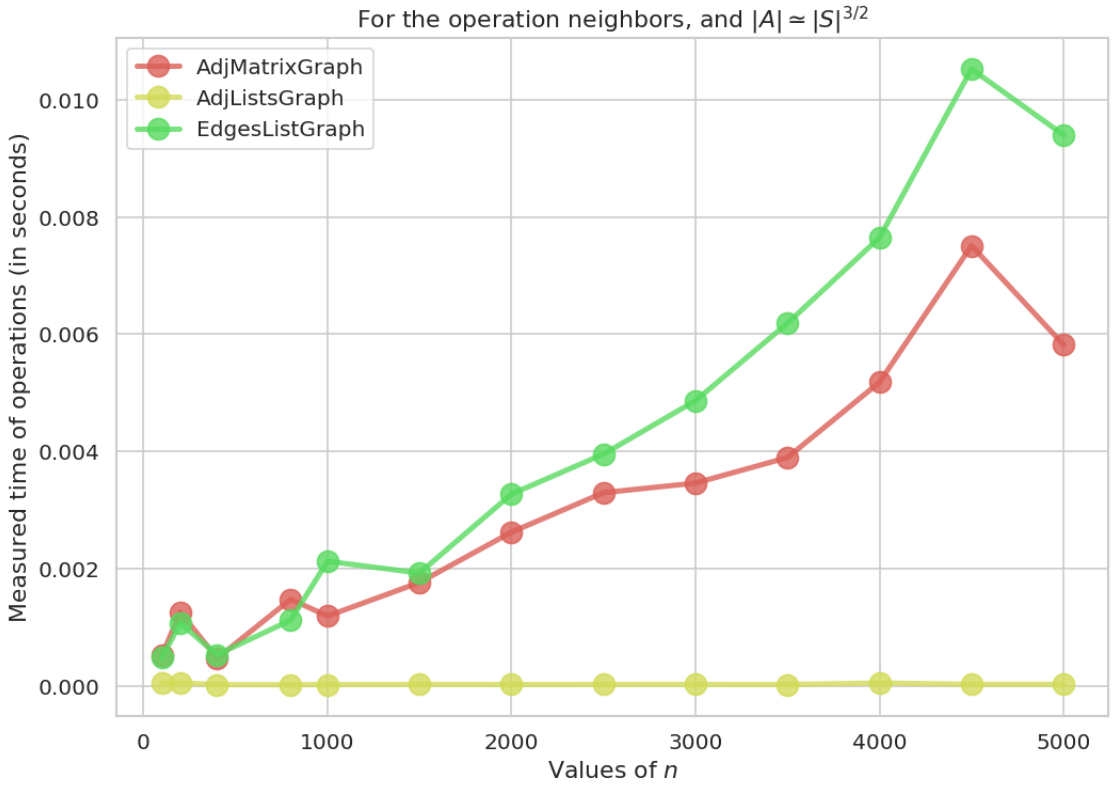

For the operation succ, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.010
0.008
0.006
0.004
0.002
0.000
0
1000
2000
3000
4000
5000
Values of n

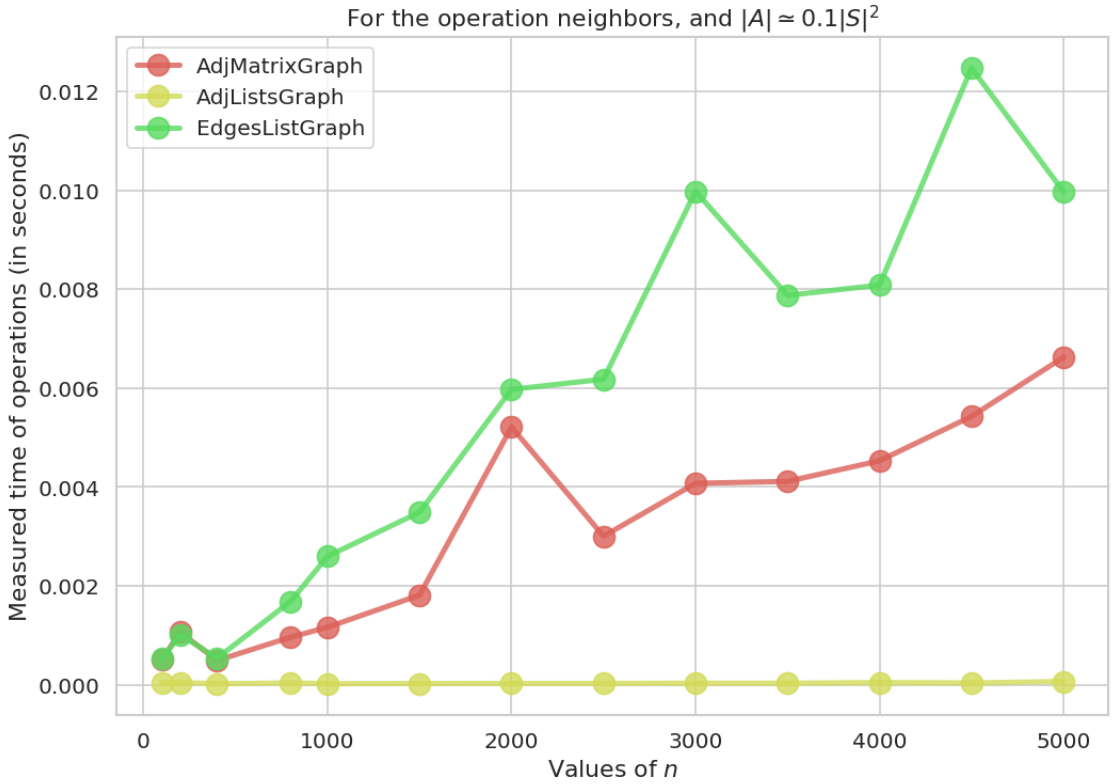

For the operation is_succ, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00012
0.00010
0.00008
0.00006
0.00004
0.00002
0
1000
2000
3000
4000
5000
Values of n

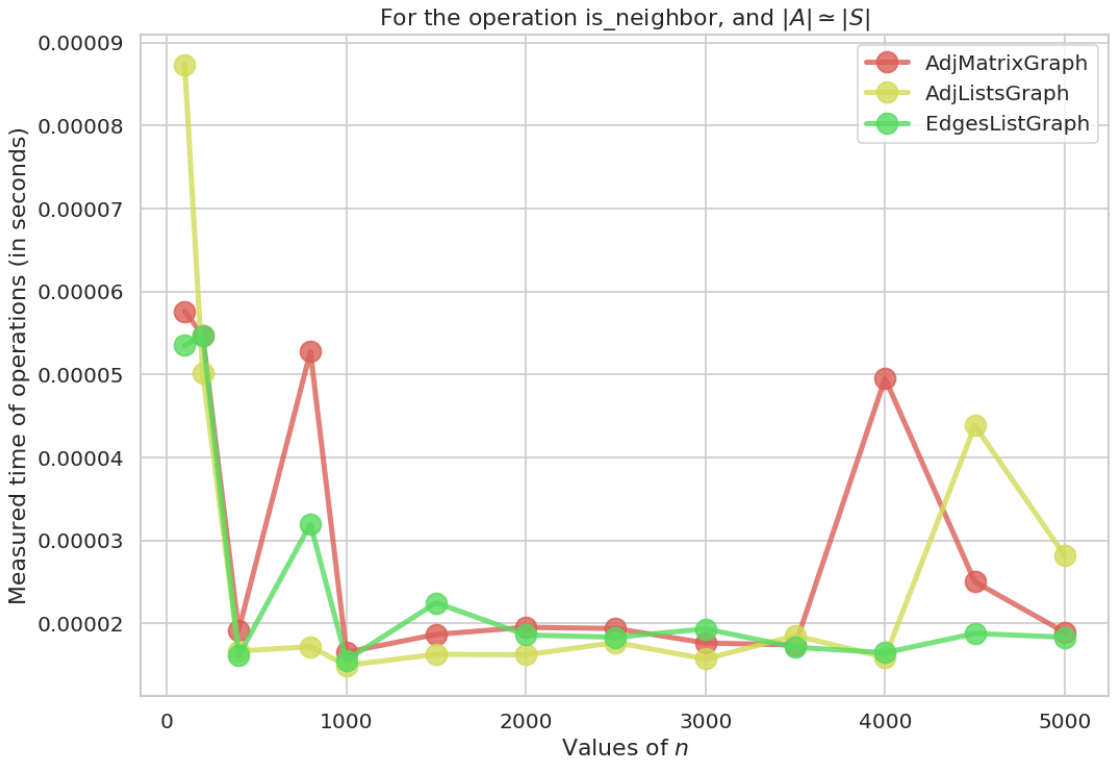

For the operation is_succ, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00006
0.00005
0.00004
0.00003
0.00002
0
1000
2000
3000
4000
5000
Values of n


For the operation is_succ, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00007
0.00006
0.00005
0.00004
0.00003
0.00002
0
1000
2000
3000
4000
5000
Values of n

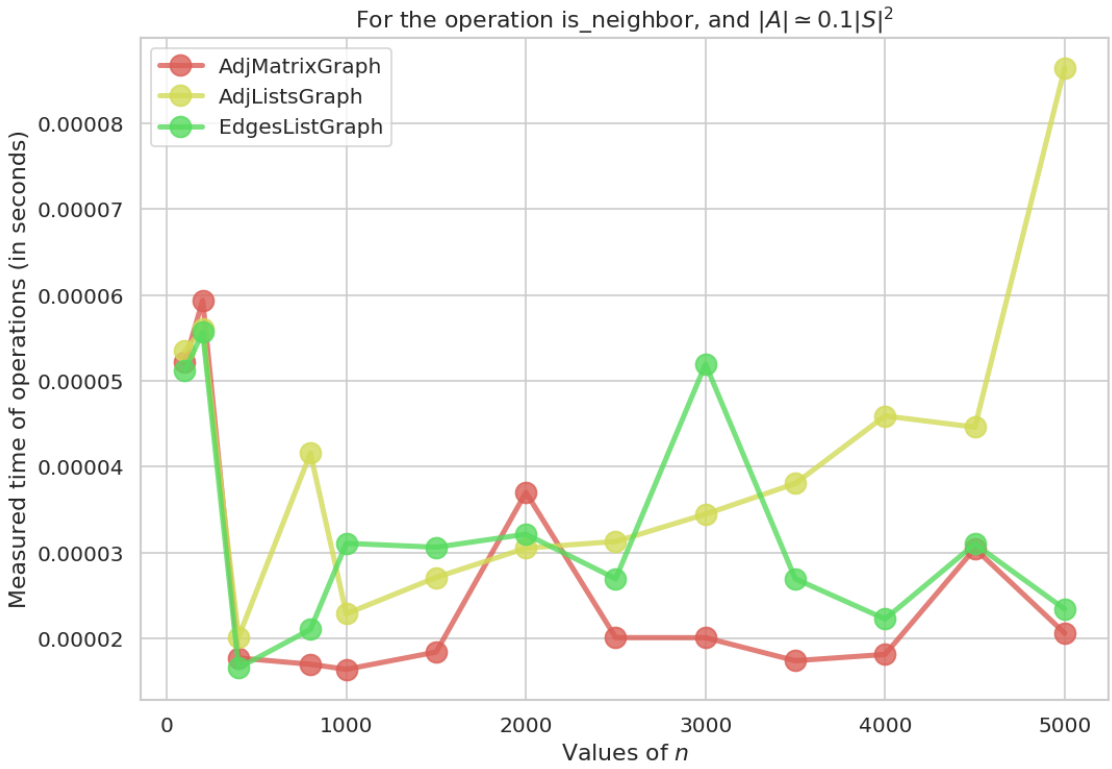

For the operation neighbors, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0
1000
2000
3000
4000
5000
Values of n

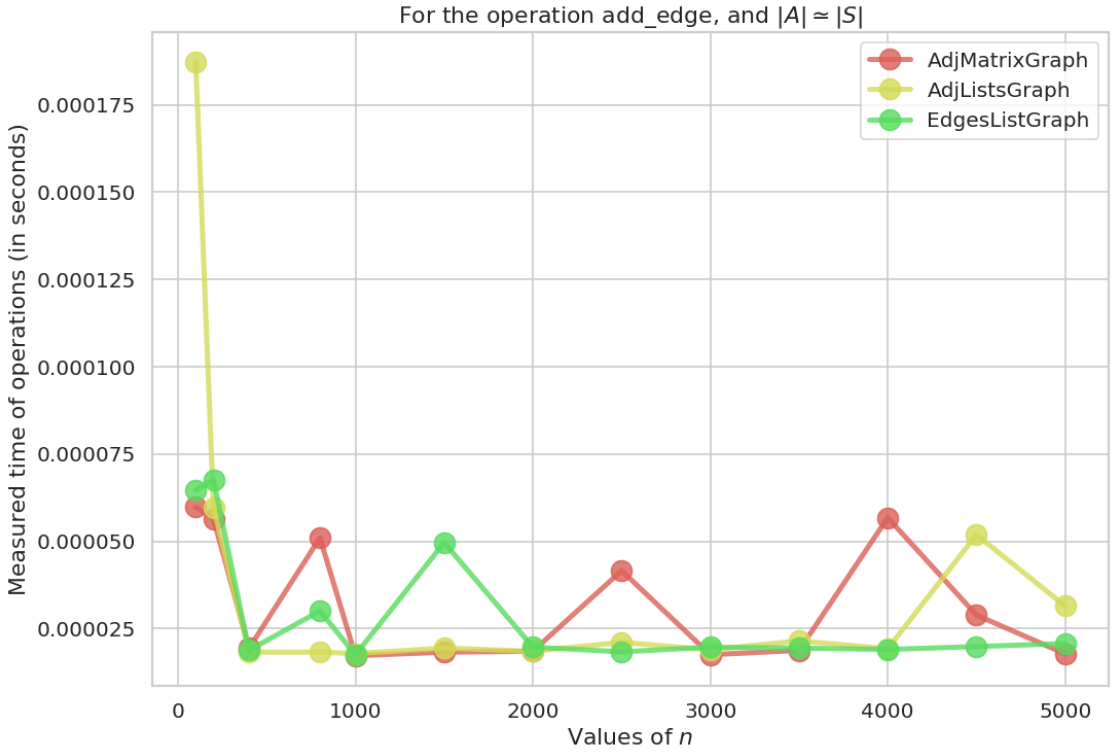

For the operation neighbors, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.000
0.002
0.004
0.006
0.008
0.010
0
1000
2000
3000
4000
5000
Values of n

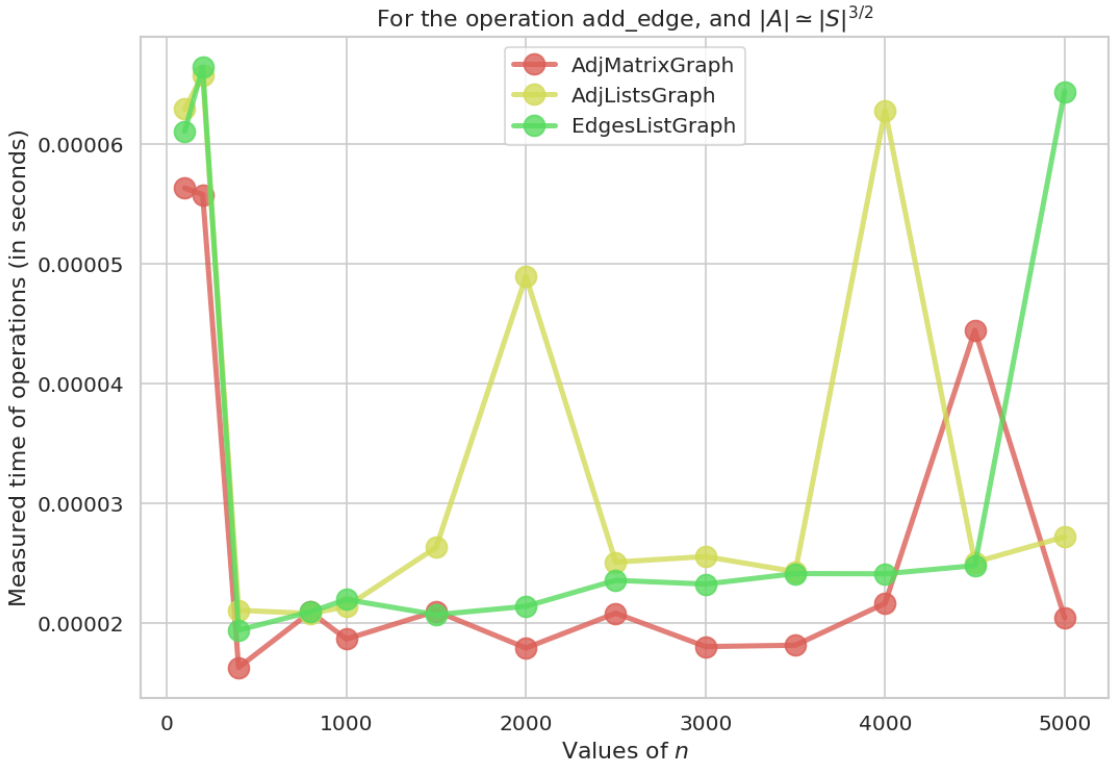

For the operation neighbors, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.000
0.002
0.004
0.006
0.008
0.010
0.012
0
1000
2000
3000
4000
5000
Values of n

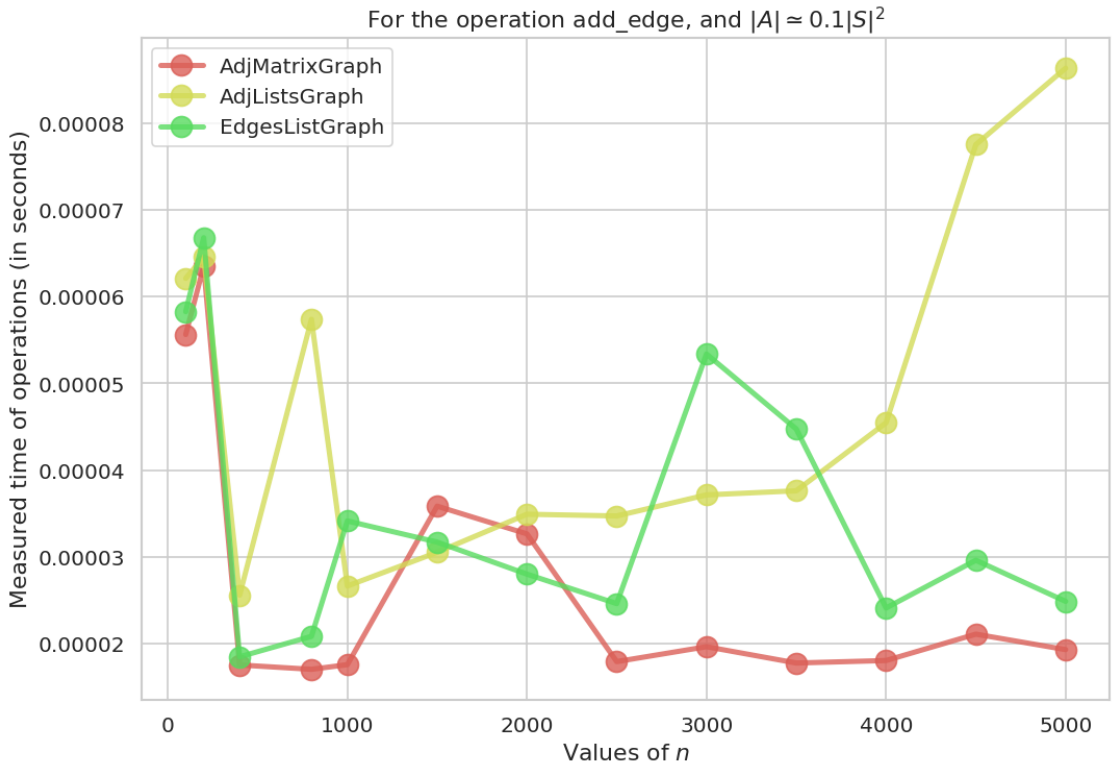

For the operation is_neighbor, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00002
0.00003
0.00004
0.00005
0.00006
0.00007
0.00008
0.00009
0
1000
2000
3000
4000
5000
Values of n


For the operation is_neighbor, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00010
0.00008
0.00006
0.00004
0.00002
0
1000
2000
3000
4000
5000
Values of n


For the operation is_neighbor, and $|A| \simeq 0.1|S|^2$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00008
0.00007
0.00006
0.00005
0.00004
0.00003
0.00002
0
1000
2000
3000
4000
5000
Values of n


For the operation add_edge, and $|A| \simeq |S|$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.000175
0.000150
0.000125
0.000100
0.000075
0.000050
0.000025
0
1000
2000
3000
4000
5000
Values of n


For the operation add_edge, and $|A| \simeq |S|^{3/2}$
AdjMatrixGraph
AdjListsGraph
EdgesListGraph
Measured time of operations (in seconds)
0.00006
0.00005
0.00004
0.00003
0.00002
0
1000
2000
3000
4000
5000
Values of n



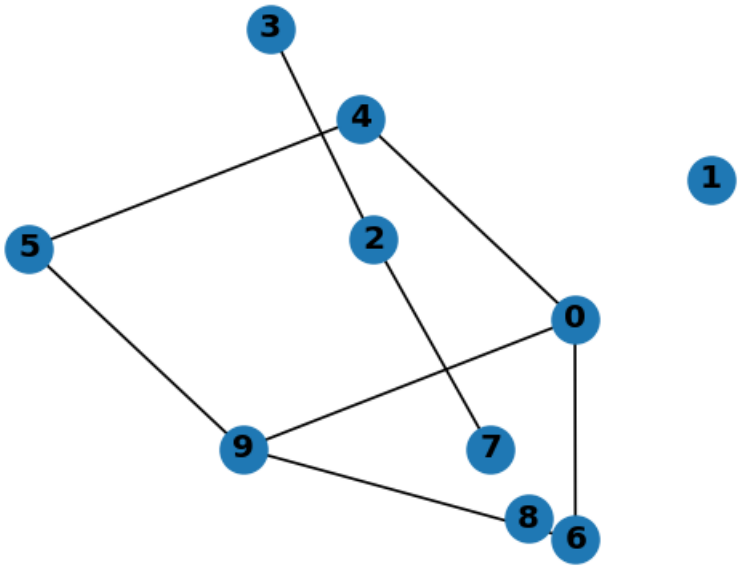
3.4 Parcours en profondeur

On va simplement implémenter l'algorithme donné en cours, avec deux fonctions génériques `post_visit` et `pre_visit`.

3.4.1 Version récursive : vue en cours

Un exemple :

```
In [219]: random.seed(12)
          graph = randomGraph(AdjMatrixGraph, 10, 0.05, oriented=False)

In [220]: plt.figure(figsize=(4, 3))
          graph.draw()

Out[220]: <Figure size 480x360 with 0 Axes>

/usr/local/lib/python3.6/dist-packages/networkx/drawing/nx_pylab.py:579: MatplotlibDeprecationW
The iterable function was deprecated in Matplotlib 3.1 and will be removed in 3.3. Use np.itera
  if not cb.iterable(width):
```




```
In [221]: dfs_recursive(graph, 0)
```

```
Previsit of u = 0
  Previsit of u = 4
    Previsit of u = 5
      Previsit of u = 9
        Previsit of u = 6
        Postvisit of u = 6
      Postvisit of u = 9
    Postvisit of u = 5
  Postvisit of u = 4
Postvisit of u = 0
```

```
Out[221]: [True, False, False, False, True, True, True, False, False, True]
```

Ici on a pu vérifier que sur cet exemple de graphe, $0, 4, 5, 9, 6$ sont dans la même composante connexe.

3.4.2 Version itérative : pas vue en cours, avec une pile

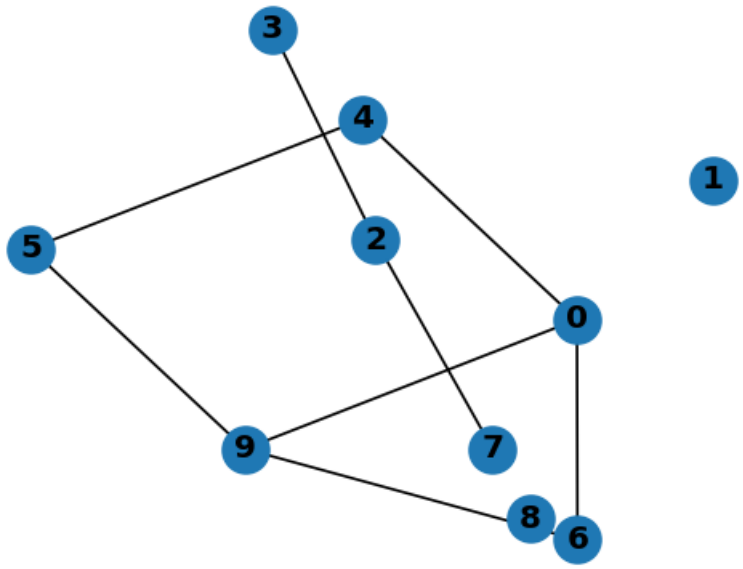
```
In [228]: def pre_visit(u):
              print(f"Previsit of u = {u}")
```

```
In [229]: def post_visit(u):
              print(f"Postvisit of u = {u}")
```

```
In [230]: def dfs_iterative(graph, start, seen=None):
              """ DFS, detect connected component, iterative implementation.

              - graph: directed graph (any of the class defined above)
              - node: from where start graph exploration
              - seen (bool array): will be set true for the connected component containing nod

              - Complexity: O(|S|+|A|).
              """
              if seen is None:
                  seen = [False for _ in range(graph.nb_vertexes)]
              seen[start] = True
              to_visit_next = [start]
              while to_visit_next:   # while stack is not empty
                  node = to_visit_next.pop()  # head of the stack / tête de la pile, O(1)
                  pre_visit(node)
                  for neighbor in graph.neighbors(node):
                      if not seen[neighbor]:
                          seen[neighbor] = True
                          to_visit_next.append(neighbor)  # add to the stack, O(1)
                  post_visit(node)  # /!\ not the same order as for the recursive function!
              return seen
```

Un exemple :

```
In [231]: random.seed(12)
          graph = randomGraph(AdjMatrixGraph, 10, 0.05, oriented=False)

In [234]: plt.figure(figsize=(4, 3))
          graph.draw()

Out[234]: <Figure size 480x360 with 0 Axes>
```



```
In [235]: dfs_iterative(graph, 0)

Previsit of u = 0
Postvisit of u = 0
Previsit of u = 9
Postvisit of u = 9
Previsit of u = 5
Postvisit of u = 5
Previsit of u = 6
Postvisit of u = 6
Previsit of u = 4
Postvisit of u = 4


Out[235]: [True, False, False, False, True, True, True, False, False, True]
```

Ici on a pu vérifier que sur cet exemple de graphe, $0, 4, 5, 9, 6$ sont dans la même composante connexe.

3.5 Application : composantes connexes d'un graphe non orienté

```
In [261]: def find_connected_components(graph):
              """ Find all the connected components of a graph.

              - graph: undirected graph (any of the class defined above)
              - returns: list of vertices in a cycle, or None

              - Complexity: O(|S|+|A|).
              """
              n = graph.nb_vertexes
              seen = [False for _ in range(n)]
              # all nodes start by having their unique connected components
              representants = [i for i in range(n)]  # maps i to a representant of its connect
              start = -1
              while not all(seen):
                  start += 1
                  if seen[start]:
                      continue
                  seen[start] = True
                  to_visit_next = [start]
                  while to_visit_next:   # while stack is not empty
                      node = to_visit_next.pop()  # head of the stack / tête de la pile, O(1)
                      for neighbor in graph.neighbors(node):
                          if not seen[neighbor]:
```

```
                    seen[neighbor] = True
                    representants[neighbor] = start
                    to_visit_next.append(neighbor)  # add to the stack, O(1)
        # now we can build the list of set of all connected components
        list_of_connected_components = [
            { i for i in range(n) if representants[i] == representant }
            for representant in set(representants)
        ]
        return list_of_connected_components, representants
```

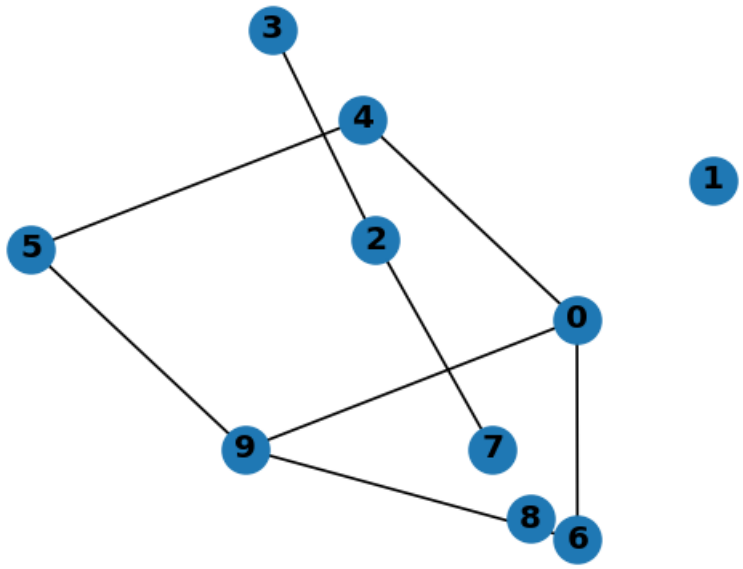
Un exemple :

```
In [262]: random.seed(12)
          graph = randomGraph(AdjMatrixGraph, 10, 0.05, oriented=False)
```

```
In [263]: plt.figure(figsize=(4, 3))
          graph.draw()
```

```
Out[263]: <Figure size 480x360 with 0 Axes>
```

```
/usr/local/lib/python3.6/dist-packages/networkx/drawing/nx_pylab.py:579: MatplotlibDeprecationW
The iterable function was deprecated in Matplotlib 3.1 and will be removed in 3.3. Use np.itera
  if not cb.iterable(width):
```




```
In [264]: find_connected_components(graph)
```

```
Out[264]: ([{0, 4, 5, 6, 9}, {1}, {2, 3, 7}, {8}], [0, 1, 2, 2, 0, 0, 0, 2, 8, 0])
```

Ici on a pu vérifier que sur cet exemple de graphe, $0, 4, 5, 9, 6$ sont dans la même composante connexe, que 1 et 8 sont isolés, et que $2, 3, 7$ sont dans une dernière composante connexe.

3.6 Application : trouver un cycle dans un graphe non orienté

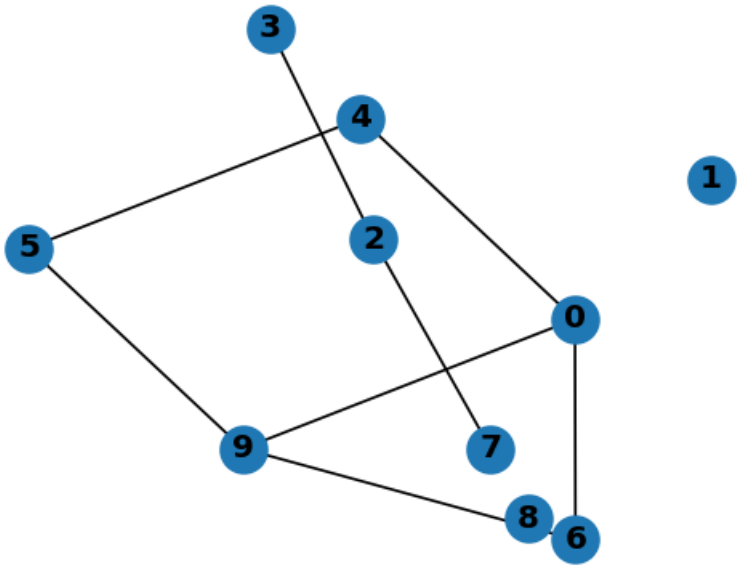
```
In [269]: def find_cycle(graph):
              """ Find a cycle in an undirected graph

              - graph: undirected graph (any of the class defined above)
              - returns: list of vertices in a cycle, or None

              - Complexity: O(|S|+|A|).
              """
              n = graph.nb_vertexes
              prec = [None] * n  # ancestor marks for visited vertices
              for u in range(n):
                  if prec[u] is None:  # unvisited vertex
                      to_visit_next = [u]  # start new DFS
                      prec[u] = u  # mark root (not necessary for this algorithm)
                      while to_visit_next:
                          u = to_visit_next.pop()
                          for v in graph.neighbors(u):  # for all neighbors
                              if v != prec[u]:  # except arcs to father in DFS tree
                                  if prec[v] is not None:
                                      cycle = [v, u]  # cycle found, (u,v) back edge
                                      while u not in (prec[v], prec[u]):  # directed
                                          u = prec[u]  # climb up the tree
                                          cycle.append(u)
                                      return cycle
                                  else:
                                      prec[v] = u  # v is new vertex in tree
                                      to_visit_next.append(v)
```

Un exemple :

```
In [270]: random.seed(12)
          graph = randomGraph(AdjMatrixGraph, 10, 0.05, oriented=False)
```

```
In [271]: plt.figure(figsize=(4, 3))
          graph.draw()
```

```
Out[271]: <Figure size 480x360 with 0 Axes>
```




```
In [272]: find_cycle(graph)
```

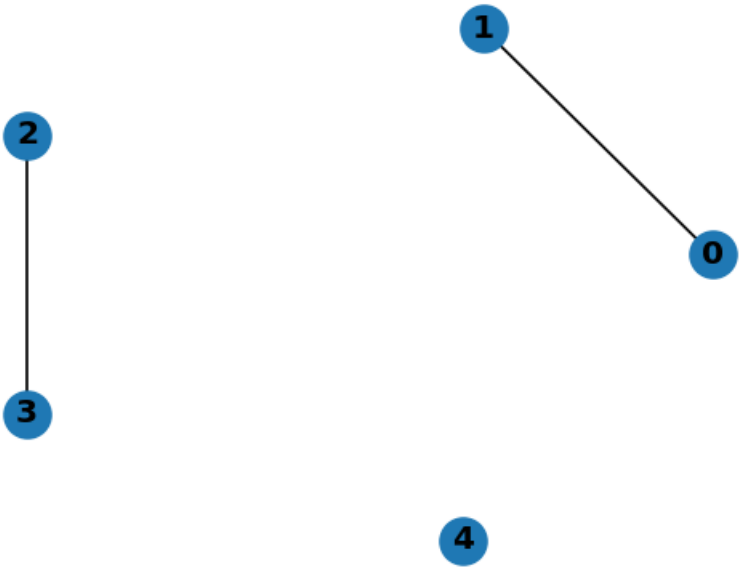
```
Out[272]: [6, 9, 0]
```

Ici on a pu vérifier que sur cet exemple de graphe, $[6, 9, 0]$ est bien un cycle.

```
In [282]: random.seed(123)
          graph = randomGraph(AdjMatrixGraph, 5, 0.05, oriented=False)
```

```
In [283]: plt.figure(figsize=(4, 3))
          graph.draw()
```

```
Out[283]: <Figure size 480x360 with 0 Axes>
```



```
In [284]: find_cycle(graph)
```

Pas de cycle dans ce graphe qui est trop creux !

```
In [ ]:
```

```
In [198]: def pre_visit(u, depth=0):
              print(f"{'  '*depth}Previsit of u = {u}")
```

```
In [199]: def post_visit(u, depth=0):
              print(f"{'  '*depth}Postvisit of u = {u}")
```

```
In [218]: def dfs_recursive(graph, node, seen=None, depth=0):
              """ DFS, detect connected component, recursive implementation.

              - graph: directed graph (any of the class defined above)
              - node: from where start graph exploration
              - seen (bool array): will be set true for the connected component containing nod

              - Complexity: O(|S|+|A|).
              """
              if seen is None:
                  seen = [False for _ in range(graph.nb_vertexes)]
              if seen[node]:
                  return seen  # nothing to do
              seen[node] = True
              pre_visit(node, depth=depth)
              for neighbor in graph.neighbors(node):
                  if not seen[neighbor]:
                      dfs_recursive(graph, neighbor, seen=seen, depth=depth+1)
              post_visit(node, depth=depth)
              return seen
```

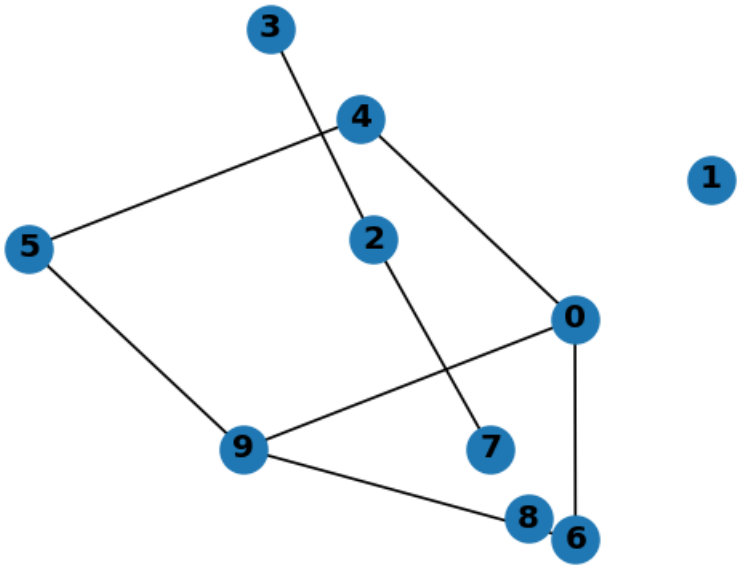
Un exemple :

```
In [219]: random.seed(12)
          graph = randomGraph(AdjMatrixGraph, 10, 0.05, oriented=False)
```

```
In [220]: plt.figure(figsize=(4, 3))
          graph.draw()
```

```
Out[220]: <Figure size 480x360 with 0 Axes>
```

```
/usr/local/lib/python3.6/dist-packages/networkx/drawing/nx_pylab.py:579: MatplotlibDeprecationW
The iterable function was deprecated in Matplotlib 3.1 and will be removed in 3.3. Use np.itera
  if not cb.iterable(width):
```




```
In [221]: dfs_recursive(graph, 0)
```

```
Previsit of u = 0
  Previsit of u = 4
    Previsit of u = 5
      Previsit of u = 9
        Previsit of u = 6
        Postvisit of u = 6
      Postvisit of u = 9
    Postvisit of u = 5
  Postvisit of u = 4
Postvisit of u = 0
```

```
Out[221]: [True, False, False, False, True, True, True, False, False, True]
```

Ici on a pu vérifier que sur cet exemple de graphe, $0, 4, 5, 9, 6$ sont dans la même composante connexe.

3.7 Parcours en largeur

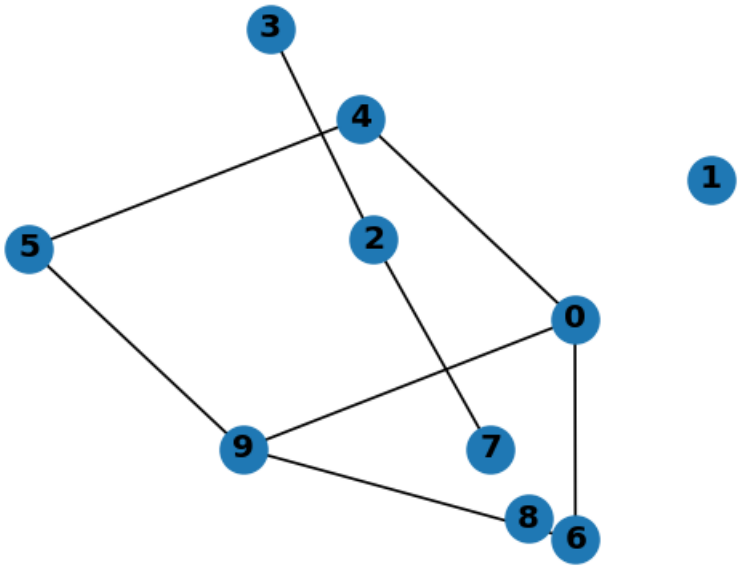
On va simplement implémenter l'algorithme donné en cours, avec deux fonctions génériques `post_visit` et `pre_visit`. On utilise une file implémentée naïvement avec un `list` de Python, mais on pourrait aussi utiliser la librairie standard de Python (`queue.Queue`).

```
In [286]: def pre_visit(u):
              print(f"Previsit of u = {u}")
```

```
In [287]: def post_visit(u):
              print(f"Postvisit of u = {u}")

In [288]: def bfs_iterative(graph, start, seen=None):
              """ DFS, iterative implementation.

              - graph: directed graph (any of the class defined above)
              - node: from where start graph exploration
              - seen (bool array): will be set true for the connected component containing nod

              - Complexity: O(|S|+|A|).
              """
              if seen is None:
                  seen = [False for _ in range(graph.nb_vertexes)]
              seen[start] = True
              to_visit_next = [start]
              while to_visit_next:   # while queue is not empty
                  node = to_visit_next.pop()  # head of the queue / tête de la file, O(1)
                  pre_visit(node)
                  for neighbor in graph.neighbors(node):
                      if not seen[neighbor]:
                          seen[neighbor] = True
                          to_visit_next.insert(0, neighbor)  # add to the queue, O(1) if well
                  post_visit(node)  # /!\ not the same order as for the recursive function!
              return seen
```

3.7.1 Un exemple :

```
In [290]: random.seed(12)
          graph = randomGraph(AdjMatrixGraph, 10, 0.05, oriented=False)

In [291]: plt.figure(figsize=(4, 3))
          graph.draw()

Out[291]: <Figure size 480x360 with 0 Axes>

/usr/local/lib/python3.6/dist-packages/networkx/drawing/nx_pylab.py:579: MatplotlibDeprecationW
The iterable function was deprecated in Matplotlib 3.1 and will be removed in 3.3. Use np.itera
  if not cb.iterable(width):
```




```
In [292]: bfs_iterative(graph, 0)
```

```
Previsit of u = 0
Postvisit of u = 0
Previsit of u = 4
Postvisit of u = 4
Previsit of u = 6
Postvisit of u = 6
Previsit of u = 9
Postvisit of u = 9
Previsit of u = 5
Postvisit of u = 5
```

```
Out[292]: [True, False, False, False, True, True, True, False, False, True]
```

On voit que 5 a été visité en dernier, c'est bien différent du parcours en profondeur qui allait le visiter avant.

```
In [293]: dfs_iterative(graph, 0)
```

```
Previsit of u = 0
Postvisit of u = 0
Previsit of u = 9
Postvisit of u = 9
Previsit of u = 5
Postvisit of u = 5
Previsit of u = 6
```

```
Postvisit of u = 6
Previsit of u = 4
Postvisit of u = 4
```

```
Out[293]: [True, False, False, False, True, True, True, False, False, True]
```

3.7.2 Distance des plus courts chemins avec un parcours en largeur

On suit l'algorithme vu en cours.

```
In [294]: def bfs_iterative_shortest_paths(graph, start):
              """ DFS, compute shortest paths, iterative implementation.

              - graph: directed graph (any of the class defined above)
              - node: from where start graph exploration
              - seen (bool array): will be set true for the connected component containing nod

              - Complexity: O(|S|+|A|).
              """
              color = ["white" for _ in range(graph.nb_vertexes)]
              parent = [None for _ in range(graph.nb_vertexes)]
              distance = [float('+inf') for _ in range(graph.nb_vertexes)]

              color[start] = "gray"
              distance[start] = 0

              to_visit_next = [start]
              while to_visit_next:   # while queue is not empty
                  node = to_visit_next.pop()  # head of the queue / tête de la file, O(1)
                  for neighbor in graph.neighbors(node):
                      if color[neighbor] == "white":
                          color[neighbor] = "gray"
                          parent[neighbor] = node
                          distance[neighbor] = distance[node] + 1
                          to_visit_next.insert(0, neighbor)  # add to the queue, O(1) if well
                  color[node] = "black"
              return color, parent, distance
```

```
In [295]: bfs_iterative_shortest_paths(graph, 0)
```

```
Out[295]: (['black',
            'white',
            'white',
            'white',
            'black',
            'black',
            'black',
            'white',
```

```
 'white',
 'black'],
[None, None, None, None, 0, 4, 0, None, None, 0],
[0, inf, inf, inf, 1, 2, 1, inf, inf, 1])
```

3.8 Algorithme de Dijkstra

On implémente l'algorithme comme on l'a vu en cours.

> L'implémentation de la file de priorité et de l'algorithme viennent de tryalgo, distribué sous licence MIT, comme ces notebooks.

3.8.1 Algorithme de Dijkstra naïf

On utilise un tas binaire min pour maintenir la file de priorité, mais sans pouvoir modifier la priorité d'un élément (on le fait plus bas). Le module heapq de la bibliothèque standard implémente les opérations d'ajout et d'extraction d'un tas binaire min (comme on l'a vu au CM2).

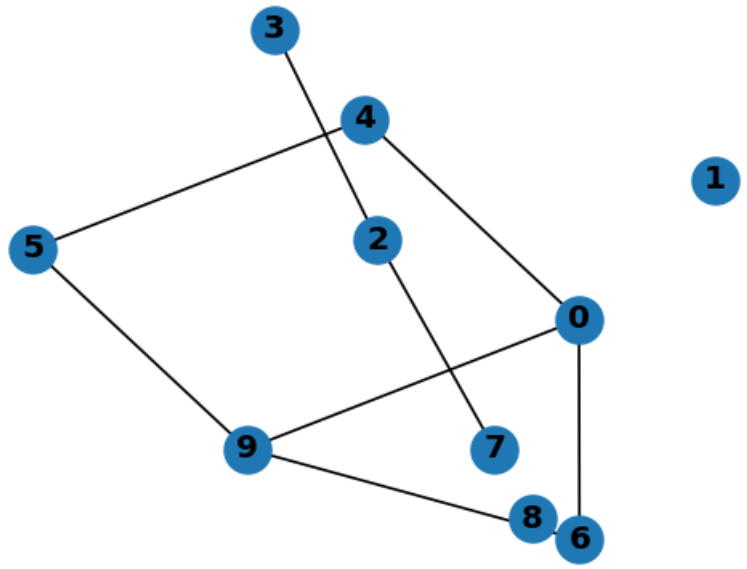
```
In [296]: from heapq import heappop, heappush
```

```
In [303]: def dijkstra(graph, weight, source=0, target=None):
              """ Single source shortest paths by Dijkstra

              - param graph: directed graph (any of the class defined above)
              - param weight: in matrix format or same listdict graph
              - assumes: weights are non-negative

              - param source: source vertex
              - param target: if given, stops once distance to target found
              - returns: distance table, precedence table

              - complexity: O(|S| + |A| log|A|)
              """
              n = graph.nb_vertexes
              assert all(weight[u][v] >= 0 for u in range(n) for v in graph.neighbors(u))
              prec = [None] * n
              black = [False] * n
              dist = [float('inf')] * n
              dist[source] = 0
              heap = [(0, source)]
              while heap:
                  dist_node, node = heappop(heap)       # Closest node from source
                  if not black[node]:
                      black[node] = True
                      if node == target:
                          break
                      for neighbor in graph.neighbors(node):
```

```
                    dist_neighbor = dist_node + weight[node][neighbor]
                    if dist_neighbor < dist[neighbor]:
                        dist[neighbor] = dist_neighbor
                        prec[neighbor] = node
                        heappush(heap, (dist_neighbor, neighbor))
        return dist, prec
```

Exemple :

```
In [304]: random.seed(12)
          graph = randomGraph(AdjMatrixGraph, 10, 0.05, oriented=False)

In [307]: plt.figure(figsize=(4, 3))
          graph.draw()

Out[307]: <Figure size 480x360 with 0 Axes>
```



Avec des poids égaux à 1 on retrouve ce que calculait le parcours en largeur :

```
In [311]: weight = np.zeros((10, 10), dtype=int)
          for (u, v) in graph.edges:
              weight[u, v] = 1

In [312]: weight

Out[312]: array([[0, 0, 0, 0, 1, 0, 1, 0, 0, 1],
                 [0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
                 [0, 0, 0, 1, 0, 0, 0, 1, 0, 0],
                 [0, 0, 1, 0, 0, 0, 0, 0, 0, 0],
```

```
                [1, 0, 0, 0, 0, 1, 0, 0, 0, 0],
                [0, 0, 0, 0, 1, 0, 0, 0, 0, 1],
                [1, 0, 0, 0, 0, 0, 0, 0, 0, 1],
                [0, 0, 1, 0, 0, 0, 0, 0, 0, 0],
                [0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
                [1, 0, 0, 0, 0, 1, 1, 0, 0, 0]])
```

```
In [313]: dijkstra(graph, weight, source=0, target=None)
```

```
Out[313]: ([0, inf, inf, inf, 1, 2, 1, inf, inf, 1],
           [None, None, None, None, 0, 4, 0, None, None, 0])
```

Mais avec des distances non triviales, le chemin optimal peut être différent.

```
In [341]: weight = np.zeros((10, 10), dtype=int)
          for (u, v) in graph.edges:
              weight[u, v] = 1 + 10*(v < 6 and u < 6)
```

```
In [342]: weight
```

```
Out[342]: array([[ 0,  0,  0,  0, 11,  0,  1,  0,  0,  1],
                 [ 0,  0,  0,  0,  0,  0,  0,  0,  0,  0],
                 [ 0,  0,  0, 11,  0,  0,  0,  1,  0,  0],
                 [ 0,  0, 11,  0,  0,  0,  0,  0,  0,  0],
                 [11,  0,  0,  0,  0, 11,  0,  0,  0,  0],
                 [ 0,  0,  0,  0, 11,  0,  0,  0,  0,  1],
                 [ 1,  0,  0,  0,  0,  0,  0,  0,  0,  1],
                 [ 0,  0,  1,  0,  0,  0,  0,  0,  0,  0],
                 [ 0,  0,  0,  0,  0,  0,  0,  0,  0,  0],
                 [ 1,  0,  0,  0,  0,  1,  1,  0,  0,  0]])
```

```
In [343]: dijkstra(graph, weight, source=0, target=None)
```

```
Out[343]: ([0, inf, inf, inf, 11, 2, 1, inf, inf, 1],
           [None, None, None, None, 0, 9, 0, None, None, 0])
```

On voit que le chemin optimal pour aller de 0 à 6 n'est pas l'arc direct $0 \to 6$, de poids 11, mais $0 \to 9 \to 6$ de poids $1 + 1 = 2$.

3.8.2 File de priorité min : implémentation maison

Le fait d'implémenter intelligement la méthode `update` ci dessous permet d'avoir un algorithme de Dijkstra qui soit efficace.

```
In [352]: class OurHeap:
              """ min heap

              * heap: is the actual heap, heap[1] = index of the smallest element
              * rank: inverse of heap with rank[x]=i iff heap[i]=x
              * n: size of the heap
```

```
    :complexity: init O(n log n), len O(1),
                 other operations O(log n) in expectation
                 and O(n) in worst case, due to the usage of a dictionary
    """
    def __init__(self, items):
        self.heap = [None]  # index 0 will be ignored
        self.rank = {}
        for x in items:
            self.push(x)

    def __len__(self):
        return len(self.heap) - 1

    def push(self, x):
        """Insert new element x in the heap.
           Assumption: x is not already in the heap"""
        assert x not in self.rank
        i = len(self.heap)
        self.heap.append(x)     # add a new leaf
        self.rank[x] = i
        self.up(i)              # maintain heap order

    def pop(self):
        """Remove and return smallest element"""
        root = self.heap[1]
        del self.rank[root]
        x = self.heap.pop()     # remove last leaf
        if self:                # if heap is not empty
            self.heap[1] = x    # put last leaf to root
            self.rank[x] = 1
            self.down(1)        # maintain heap order
        return root

    def up(self, i):
        """The value of heap[i] has decreased. Maintain heap invariant."""
        x = self.heap[i]
        while i > 1 and x < self.heap[i // 2]:
            self.heap[i] = self.heap[i // 2]
            self.rank[self.heap[i // 2]] = i
            i //= 2
        self.heap[i] = x        # insertion index found
        self.rank[x] = i

    def down(self, i):
        """the value of heap[i] has increased. Maintain heap invariant."""
        x = self.heap[i]
        n = len(self.heap)
```

```
            while True:
                left = 2 * i       # climb down the tree
                right = left + 1
                if (right < n and self.heap[right] < x and
                        self.heap[right] < self.heap[left]):
                    self.heap[i] = self.heap[right]
                    self.rank[self.heap[right]] = i   # go back up right child
                    i = right
                elif left < n and self.heap[left] < x:
                    self.heap[i] = self.heap[left]
                    self.rank[self.heap[left]] = i    # go back up left child
                    i = left
                else:
                    self.heap[i] = x   # insertion index found
                    self.rank[x] = i
                    return

        def update(self, old, new):
            """Replace an element in the heap
            """
            i = self.rank[old]      # change value at index i
            del self.rank[old]
            self.heap[i] = new
            self.rank[new] = i
            if old < new:           # maintain heap order
                self.down(i)
            else:
                self.up(i)
```

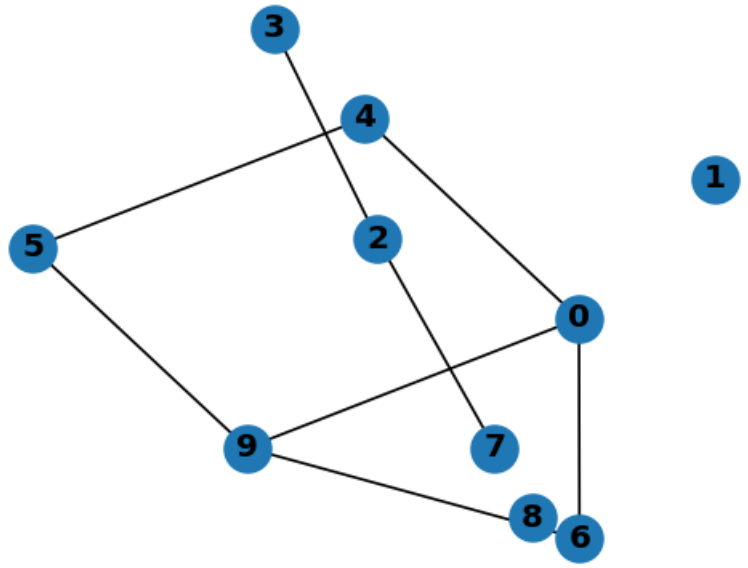
3.8.3 Algorithme de Dijkstra

```
In [353]: def dijkstra_update_heap(graph, weight, source=0, target=None):
              """ Single source shortest paths by Dijkstra
                  with a heap implementing item updates

              - param graph: directed graph (any of the class defined above)
              - param weight: in matrix format or same listdict graph
              - assumes: weights are non-negative

              - param source: source vertex
              - param target: if given, stops once distance to target found
              - returns: distance table, precedence table

              - complexity: O(|S| + |A| log|A|)
              """
              n = graph.nb_vertexes
              assert all(weight[u][v] >= 0 for u in range(n) for v in graph.neighbors(u))
              prec = [None] * n
```

```
    dist = [float('inf')] * n
    dist[source] = 0
    heap = OurHeap([(dist[node], node) for node in range(n)])
    while heap:
        dist_node, node = heap.pop()        # Closest node from source
        if node == target:
            break
        for neighbor in graph.neighbors(node):
            old = dist[neighbor]
            new = dist_node + weight[node][neighbor]
            if new < old:
                dist[neighbor] = new
                prec[neighbor] = node
                heap.update((old, neighbor), (new, neighbor))
    return dist, prec
```

Exemple :

```
In [354]: random.seed(12)
          graph = randomGraph(AdjMatrixGraph, 10, 0.05, oriented=False)

In [355]: plt.figure(figsize=(4, 3))
          graph.draw()

Out[355]: <Figure size 480x360 with 0 Axes>
```



Avec des poids égaux à 1 on retrouve ce que calculait le parcours en largeur :

```
In [356]: weight = np.zeros((10, 10), dtype=int)
          for (u, v) in graph.edges:
              weight[u, v] = 1

In [357]: dijkstra_update_heap(graph, weight, source=0, target=None)

Out[357]: ([0, inf, inf, inf, 1, 2, 1, inf, inf, 1],
           [None, None, None, None, 0, 4, 0, None, None, 0])
```

Mais avec des distances non triviales, le chemin optimal peut être différent.

```
In [358]: weight = np.zeros((10, 10), dtype=int)
          for (u, v) in graph.edges:
              weight[u, v] = 1 + 10*(v < 6 and u < 6)

In [359]: weight

Out[359]: array([[ 0,  0,  0,  0, 11,  0,  1,  0,  0,  1],
                 [ 0,  0,  0,  0,  0,  0,  0,  0,  0,  0],
                 [ 0,  0,  0, 11,  0,  0,  0,  1,  0,  0],
                 [ 0,  0, 11,  0,  0,  0,  0,  0,  0,  0],
                 [11,  0,  0,  0,  0, 11,  0,  0,  0,  0],
                 [ 0,  0,  0,  0, 11,  0,  0,  0,  0,  1],
                 [ 1,  0,  0,  0,  0,  0,  0,  0,  0,  1],
                 [ 0,  0,  1,  0,  0,  0,  0,  0,  0,  0],
                 [ 0,  0,  0,  0,  0,  0,  0,  0,  0,  0],
                 [ 1,  0,  0,  0,  0,  1,  1,  0,  0,  0]])

In [360]: dijkstra_update_heap(graph, weight, source=0, target=None)

Out[360]: ([0, inf, inf, inf, 11, 2, 1, inf, inf, 1],
           [None, None, None, None, 0, 9, 0, None, None, 0])
```

3.9 Algorithme A*

Je ne vais pas prendre le temps de l'implémenter. Allez regarder un des liens suivants, si vous avez envie :

- https://fr.wikipedia.org/wiki/Algorithme_A*
- https://www.redblobgames.com/pathfinding/a-star/implementation.html

D'autres :

- https://medium.com/@nicholas.w.swift/easy-a-star-pathfinding-7e6689c7f7b2
- https://gist.github.com/jamiees2/5531924

3.10 Conclusion

C'est bon pour aujourd'hui !