# Decomposing Minimal Observability for Transactional Services * 

Debmalya Biswas and Blaise Genest, IRISA/INRIA\&CNRS, Campus de Beaulieu, 35042 Rennes Cedex, France


#### Abstract

For complex services, logging is an integral part of many middleware aspects, especially, transactions and monitoring. In the event of a failure, the log allows us to deduce the cause of failure (diagnosis), recover by compensating the logged actions (atomicity), etc. However, for heterogeneous services, logging all the actions is often impracticable due to privacy/security constraints. Also, logging is expensive in terms of both time and space. Thus, we are interested in determining the absolute minimal number of actions that needs to be logged, to know with certainty the actual sequence of executed actions from any given partial log. This problem happens to be NP-Complete. We propose a decomposition framework in order to use a divide and conquer algorithm. This method dramatically decreases the complexity for hierarchical services (up to 2 exponentials) and can also be used in distributed services.


## 1 Introduction

An interesting problem for complex systems is to determine a minimal set of actions that needs to be observable such that a given property holds. Some of the properties studied in literature of discrete event systems are normality [11], observability [10], state observability [14], diagnosability [18], etc. Our system corresponds to a (composite) Web service. A Web service [1] refers to an online service accessible via Internet standard protocols. A composite service, composed of already existing (component) services, combines the capabilities of its components to provide a new service. A service schema which specifies the execution order of its components, can be modeled as a Finite State Machine (FSM), performing actions on global variables. We do not tackle here the transformation of a service into a FSM, which should be handled with care to yield a FSM of reasonable size (see [19] and example 1).

Our long-term objective is to provide a transactional framework for (composite) Web services. A transaction

[^0]can be considered as a group of actions encapsulated by the operations Begin and Commit/Abort, having the following properties: Atomicity (A), Consistency (C), Isolation (I) and Durability (D). Here, we focus on the atomicity aspect, that is, either all the actions of a transaction are executed or none. In the event of a failure, atomicity is preserved by compensation [4]. Compensation consists of executing the compensating actions, corresponding to each executed action of the failed process, in reverse order of the original execution. Thus, for compensation to be feasible, we need to reconstruct each executed action or the complete history of any execution. To achieve that, we maintain a log of observable actions. In addition to the obvious space overhead of logging (in our testing, about 4 times smaller), the complete log may not always be accessible. For a composite service, the providers of its component services are different. As such, their privacy/security constraints may prevent them from exposing (part of) the logs corresponding to the execution at their sites. Also, heterogeneity may lead to the logs being maintained in different formats, rendering some of them incomprehensible. Hence, we want from such a partial log to know with certainty the actual sequence of executed actions, to be able to compensate it.

Section 2 introduces the required formal preliminaries including the precise problem statement. Clearly, we are interested in logging the smallest number of actions possible. However, it appears that determining the minimal number of actions to log, such that any execution of a system is compensable, is NP-Complete. This is not very surprising, since determining the minimal number of actions needed to achieve a property is usually NP-Complete, e.g., for sensor selections [20, 13]. What is more surprising is that it is NP-Complete even with strong restrictions on the graph (see section 3). Also, the problem cannot be approximated [16] in polynomial time.

In order to compute the minimal number of actions to log in large systems, we develop a decomposition framework. Intuitively, the algorithm we propose in section 4, first decomposes the FSM into smaller components. The problem is that even if the components are simple (with only one input and output), the union of the minimal sets of actions of different components is not necessarily an observable set
of actions for the original FSM. One solution could be to resort to function summarization, but then only an overapproximation of the needed set of actions is obtained. Nevertheless, we show that it suffices to run the algorithm with slightly different parameters on each component, a number of times which depends only on the number of inputs and outputs of the components (section 5.2). That is, a fixed number of times for simple components (or for components with few inputs and outputs). We thus obtain a divide and conquer algorithm. We present a complexity analysis using the brute force method on each component which illustrates the benefit of our method, but we can use any other algorithm to compute the minimal observability, as [8]. Preliminary experiments (section 4.1) reveal that simple decompositions usually do not allow for better scaling for randomly generated graphs. However, our algorithm decomposes very efficiently the real-life examples we found into small simple components. Our implementations and examples are available at http://www.crans.org/-genest/dns/.

Interestingly, our algorithm can be used for distributed systems (section 5.3), and also in large graphs with a small compressed representation, as shown in section 5.1. We use the standard hierarchical system to depict this compressed representation, as is often used for words [15], Finite State Machines [2], and even trees [12]. For words, e.g., hierarchical structures correspond to the LZ compression [15]. Here, the complexity can be up to two exponentials better using our decomposition method than without.

## 2 Preliminaries

Formally, we model a transactional service as a finite state machine, that is, a 4-tuple $M=\left(Q, s_{0}, s_{f}, \mathcal{T}\right)$, where:

- $Q$ is the finite set of states,
- $s_{0}$ and $s_{f}$ are the initial and final states, respectively,
- $\mathcal{T} \subseteq Q \times Q$ is the (partial) transition relation.

We describe our FSMs as graphs with a unique input and output point, each node and arc corresponds to a state and transition, but we ignore the alphabet. We assume that the service $M$ does not have any unreachable states and that al1 states can reach the final state $s_{f}$. For convenience, we also assume that there are no outgoing edges from $s_{f}$ and no incoming edges to $s_{0}$. ${ }^{1} \mathrm{We}$ say that an execution sequence $\rho=\tau_{1} \cdots \tau_{n} \in \mathcal{T}^{*}$ is a path of $M$ if there exists $q_{0}, \cdots, q_{n} \in Q^{n+1}$ with $\tau_{i}=\left(q_{i-1}, q_{i}\right)$ for all $1 \leq i \leq n$. A path is called initial if furthermore $q_{0}=s_{0}$. We denote by $\mathcal{P}(M)$ the set of initial paths in $M$. Finally, we denote by $|M|$ the size of $M$, that is, its number of states.

[^1]In general, for any execution $\rho$, we call observation projections the the observation we have after $\rho$ was executed (a sequence of actions, control points, data ... ). We say that an observable projection $\sigma$ is uncertain if there exists two paths having the same projection. The FSM $M$ is execution sequence detectable iff none of its observable projections are uncertain.

Definition 1 For an FSM $M$, let $\mathcal{T}_{O} \subseteq \mathcal{T}$ be the set of observable transitions. The observation projection $\mathrm{Obs}_{O}$ : $\mathcal{T}^{*} \longrightarrow \mathcal{T}_{O}^{*}$ is the morphism with $\operatorname{Obs}_{O}(a)=a$ if $a \in \mathcal{T}_{O}$, and $\operatorname{Obs}_{O}(a)=\epsilon$ if $a \in \mathcal{T} \backslash \mathcal{T}_{O}$, with $\epsilon$ the empty word.

That is, $\operatorname{Obs}_{O}(\rho)$ is the subsequence of $\rho$ obtained by eliminating from $\rho$ every occurence of a tuple which is not in $\mathcal{T}_{O}$. With such an observation projection $\mathrm{Obs}_{O}$, the only way of having execution sequence detectability is to have every transition observable. Indeed, as soon as there exists even one non-observable transition, the service is not execution sequence detectable. Else, let us take a path $\rho \tau$ with the last transition $\tau \notin \mathcal{T}_{O}$. Then, $\operatorname{Obs}_{O}(\rho \tau)=\operatorname{Obs}_{O}(\rho)$. A usual way to overcome such a problem is to ask for certainty only up to the last few events of the sequence [14]. However, this turnaround does not make sense in our framework since if we cannot compensate the very last action, then we cannot compensate any action at all. As such, we design a new observation mechanism, where the last control point reached before failure is monitored, even if the last action is not logged. In practice, it means that every state that is reached is monitored, and overstack the previous state in a special memory buffer.

Definition 2 Let $M$ be an $F S M, \mathcal{T}_{O} \subseteq \mathcal{T}$. The observation projection $\mathrm{Obs}_{O}^{\text {last }}: \mathcal{T}^{*} \longrightarrow\left(\mathcal{T}_{O}^{*}, Q\right)$ is the function $\operatorname{Obs}_{O}^{\text {last }}(\rho)=\left(\operatorname{Obs}_{O}(\rho), q\right)$ for all $\rho \in \mathcal{P}(M)$ ending in $q$.

We will stick with this definition of observability for the rest of the paper. As mentioned before, we are interested in logging as few transitions as possible.

Problem statement. Given an FSM $M=\left(Q, s_{0}, s_{f}, \mathcal{T}\right)$, we call $\mathcal{T}_{O}$ an observable set of transitions if the service is execution sequence detectable with $\mathrm{Obs}_{O}^{\text {last }}$. We want to determine a minimal observable set of transitions $\mathcal{T}_{O} \subseteq \mathcal{T}$.

The cardinality of such a minimal observable set $\mathcal{T}_{O}$ of an FSM $M$ is referred to as its observable size $M O(M)=$ $\left|\mathcal{T}_{O}\right|$. Notice that as is usual with decision and computation algorithms, it is sufficient to have an algorithm which from an FSM gives its observable size. That is, we can derive a minimal observable set of the FSM based on knowledge of its observable size in polynomial time.


Figure 1. Travel funds request workflow.

Example 1 We consider in Fig. 1 a travel funds request service, inspired by the workflow in [17]. It involves different departments across organizations.

We model the service using the FSM $M=\left(S, s_{0}, s_{f}, \mathcal{T}\right)$ representation, as shown in Fig. 2. Notice that this FS$M$ is a simplification of the service, since for instance the choice between the team leader or supervisor approvals is not represented. The reason is that they are both associated with an empty compensating transition, hence knowing which path was taken here is not necessary to be able to perform recovery. However, it is necessary to know which bank issued the cheque in order to able to compensate it, by a "Cancel Last American Express (Citibank) Cheque". It is also possible to handle data being written to the database. For instance, if there is no "Cancel Last Cheque" mechanism, it is possible to force the transition "Update Accounts Database" to be observable, which would lead to the exact amount of the cheque being written to the log, and recovery would manually credit the amount of money written in the log to the corresponding account.

Now, let $\mathcal{T}_{O}=\left\{e_{2}, e_{3}\right\}$ and a failure occurs while processing $e_{8}$, that is, the cheque is not issued or delivered correctly. Then, Obs $_{O}^{\text {last }}\left(e_{1} e_{2} e_{5} e_{7}\right)=\left(e_{2}, s_{5}\right)=$ $\mathrm{Obs}_{O}^{\text {last }}\left(e_{1} e_{2} e_{4} e_{6} e_{7}\right)$. Thus, we do not know if an American Express or Citibank cheque was processed. With $\mathcal{T}_{O}^{\prime}=\left\{e_{2}, e_{6}\right\}$, we have Obs $s_{O}^{\text {last }}\left(e_{1} e_{2} e_{5} e_{7}\right)=\left(e_{2}, s_{5}\right)$ and $\operatorname{Obs}_{O}^{\text {last }}\left(e_{1} e_{2} e_{4} e_{6} e_{7}\right)=\left(e_{2} e_{6}, s_{5}\right)$, and $\mathcal{T}_{O}^{\prime}$ is an observable set of transitions. Notice that every path from $s_{0}$ to $s_{f}$ uses $\mathcal{T}_{O}^{\prime}$.


Figure 2. FSM representation of Fig. 1.

## 3 A Difficult Problem

We first relate the problem of computing $M O(M)$ using our definition of observable projections with other known problems. We state now that computing the minimal observable set is equivalent to the uniconnected subgraph problem, also called the minimal marker placement problem [13], in the meaning of the following proposition.

Proposition 1 Let $M$ be an $F S M$ and $\mathcal{T}_{O}$ a subset of transitions of $M$. Denote by $M^{\prime}$ the FSM $M$ obtained by deleting all transitions belonging to $\mathcal{T}_{O}$. Then, $\mathcal{T}_{O}$ is an observable set of $M$ iff there does not exist a pair of paths $\rho_{1} \neq \rho_{2}$ of $M^{\prime}$ with $\rho_{1}$ beginning and ending at the same states as $\rho_{2}$.

To prove the proposition 1, it suffices to prove that if there does not exist a pair of paths $\rho_{1} \neq \rho_{2}$ of $M^{\prime}$ with $\rho_{1}$ beginning and ending at the same states as $\rho_{2}$, then from any observable projection $\left(\sigma, q_{n+1}\right)$, we can reconstruct in a unique way a path with $\operatorname{Obs}_{O}^{\text {last }}(\rho)=\left(\sigma, q_{n+1}\right)$. The converse is trivial. Indeed, it suffices to define the only path $\rho_{i}$ of $M^{\prime}$ between $q_{i}^{\prime}$ and $q_{i+1}$ for $\sigma=\left(q_{i}, q_{i}^{\prime}\right)_{i=1}^{n}$, and $i=0 \cdots n$ (we fix $q_{0}^{\prime}=s_{0}$ the initial state of $M^{\prime}$, and recall that $q_{n+1}$ is the last observed state). Then, the path $\rho=\rho_{0}\left(q_{1}, q_{1}^{\prime}\right) \rho_{1} \cdots\left(q_{n}, q_{n}^{\prime}\right) \rho_{n}$ is the only path with $\pi_{O}^{\text {last }}(\rho)=\left(\sigma, q_{n+1}\right)$. The search for each path $\rho_{i}$ can be made in linear time by a simple depth first search in $M^{\prime}$.

The fact is that the marker placement problem is an NPComplete problem. The question is then to know if there is a structural subclass of graphs which has a tractable algorithm to give the minimal observable size. We know from [13] that the minimal marker placement problem is NP-Complete even for acyclic graphs. However, the proof uses a graph with unbounded (in and out) degree. We show that the problem is NP-Complete even if the graph is both acyclic and the sum of its in and outdegree bounded by 3 (that is, indegree 2 and outdegree 1 , or vice versa). The core of the proof follows the same strategy as [13], but the encoding to get a unique starting and ending point is both easier to understand and allows a lower in and outdegree.

Theorem 1 Let $M$ be an FSM, and $k$ a number. Knowing whether $M O(M) \leq k$ is NP-Complete, even if the corresponding graph is acyclic and the sum of in and outdegree of every node bounded by 3 .

Proof. Let $M$ be a system. We reduce Vertex Cover to the problem of finding a subset of transitions $\mathcal{T}_{O}$ of $M$, such that, there are no two paths $\rho_{1} \neq \rho_{2}$ beginning and ending at the same nodes, and using no transitions of $\mathcal{T}_{O}$.

Let us take an undirected graph $(V, E)$ and a number $k$. We want to know whether there is a subset $V_{O}$ of $V$ of size $\leq k$ such that for all $(v, w) \in E$, at least one of $v, w$ belongs to $V_{O}$. This problem is NP-complete even with


Figure 3. FSM $M$.
( $V, E$ ) of degree 3 . The first FSM $M$ we build has a state space $S=V_{1} \cup V_{2} \cup E_{1} \cup E_{2}$ where $V_{i}=\left\{v_{i} \mid v \in V\right\}$ and $E_{i}=\left\{e_{i} \mid e \in E\right\}$. Furthermore, for $v, w \in V$ and $e \in E$, we have transitions

1. $\left(e_{1}, v_{1}\right) \in \mathcal{T}$ iff $v \in e \operatorname{iff}\left(v_{2}, e_{2}\right) \in \mathcal{T}$
2. $\left(v_{1}, w_{2}\right) \in \mathcal{T}$ iff $v=w$.

A graphical representation of $M$ appears in Fig. 3.
Assume that there is a subset $V_{O}$ of $V$ of size $k$ such that for all $(v, w) \in E$, at least one of $v, w$ belongs to $V_{O}$. Then, defining $\mathcal{T}_{O}=\left\{\left(v_{1}, v_{2}\right) \mid v \in V_{O}\right\}$, we have that there are no two paths $\rho_{1} \neq \rho_{2}$ with $\rho_{1}$ and $\rho_{2}$ beginning and ending at the same nodes, and not using transitions of $\mathcal{T}_{O}$. By contradiction, else we would have $\rho_{1}, \rho_{2}$ both from some $e_{1} \in E_{1}$ to some $f_{2} \in E_{2}$ and not using transitions of $\mathcal{T}_{O}$. By definition of $\mathcal{T}_{O}$, it means that for $\rho_{1}$, there exists a node $v \in e, v \in f$ such that $v \notin V_{O}$. Similarly, for $\rho_{2}$ with a node $w$. Since $\rho_{1} \neq \rho_{2}$, we have that $v \neq w$; hence $e=(v, w)$ contradicts $V_{O}$ is a vertex cover.

Conversely, assume that there is a set of transitions $\mathcal{T}_{O}$ of size $k$ such that there do not exist two paths from and to the same state without using $\mathcal{T}_{O}$. We build the set of nodes $V_{O}=\left\{v \mid\left(v_{1}, v_{2}\right) \in \mathcal{T}_{O}\right\} \cup\left\{v \mid \exists e,\left(e_{1}, v_{1}\right) \in\right.$ $\left.\mathcal{T}_{O}\right\} \cup\left\{v \mid \exists e,\left(v_{2}, e_{2}\right) \in \mathcal{T}_{O}\right\}$. Clearly, $\left|V_{O}\right| \leq\left|\mathcal{T}_{O}\right|=k$. We prove now that $V$ is a vertex cover of $(V, E)$. Assume by contradiction that there exists an edge $e=(v, w)$ such that $v, w \notin V_{O}$. Then, we argue that $e_{1} v_{1} v_{2} e_{2}$ and $e_{1} w_{1} w_{2} e_{2}$ are two paths not using $\mathcal{T}_{O}$, a contradiction.

However, so far, the graph defined is not a system since it has several states with indegree 0 (the $\left(e_{1}\right)_{e \in E}$ ), and several states with outdegree 0 (the $\left.\left(e_{2}\right)_{e \in E}\right)$. Moreover, the indegree of states $\left(v_{1}\right)_{v \in V}$ and the outdegree of states $\left(v_{2}\right)_{v \in V}$ can be 3 (the degree of the undirected graph $(V, E))$. However, it is acyclic. For the degree, one can safely transform any node $v_{1}$ with 3 ingoing transitions from states $e_{1}, f_{1}, g_{1}$ by having two nodes $v_{1}, v_{1}^{\prime}$ with transitions $\left(e_{1}, v_{1}^{\prime}\right),\left(f_{1}, v_{1}^{\prime}\right),\left(v_{1}^{\prime}, v_{1}\right)$ and $\left(g_{1}, v_{1}\right)$. Hence all nodes have indegree at most 2 . The same can be done for outdegree. The size of the minimal observable set of transitions will not change with such a transformation. Actually, with such a technique, we could start from an undirected graph of any degree.

Making the graph a system is a little more involved. We use the graph $G$ from Fig. 2. It then suffices to create a balanced binary tree of transitions with root $s_{i}$ such that there are $E$ leaves. This tree has $O(2|E|)$ nodes, that we add to the system $S$ we built from $(V, E)$. The root of the tree is the unique initial node, and every leaf is connected to a node $\left(e_{1}\right)_{e \in E}$ through a copy of graph $G$. The same is done for nodes $\left(e_{2}\right)_{e \in E}$ connected through copies of $G$ to a balanced binary tree with root $s_{f}$ the unique final state. This system has $O(|V|+|E|)$ nodes, is acyclic and of total degree 3 . Now, it is easy to show that if the minimal vertex cover has $k$ vertices, then the minimal observable set of transitions is of size $k+4|E|$. Indeed, there are $2|E|$ copies of the graph $G$ each of which requires 2 observed transitions. Once these transitions are observed, the two balanced trees are totally disconnected from each other and from the first system we had built (since every path from the initial to the final state of the graph $G$ uses one of the two observed transitions), and hence we need to observe exactly $k$ more transitions. Notice that connecting directly the tree with $S$ without using $G$ would not work since it would potentially connect $s^{0}, s^{f}$ through two different paths $s^{0} \longrightarrow e_{1} \longrightarrow e_{2} \longrightarrow s^{f}$ and $s^{0} \longrightarrow f_{1} \longrightarrow f_{2} \longrightarrow s^{f}$, with $e, f \in E$.

This theorem does not mean that the problem is impossible to solve, but that it can be solved for small enough services only. For instance, the complexity of the brute force method which generates every subset of transitions and tests whether it is observable, is $O\left(2^{|M|}\right)$ for a service $M$ with $|M|$ transitions. The question is: how can we reduce the time taken to compute $M O$ for bigger services? It also implies that a structural restriction of an FSM $M$ which has a tractable algorithm to compute $M O(M)$ is at least very complicated to find.

## 4 Simple Decomposition of Graphs

The idea we use is that of divide and conquer: we would like to decompose the given graph into two parts, and compute $M O$ independently on each part. Since the best known algorithm takes exponential time (adding even one node to a graph can make the algorithm twice as slow), even an unfair decomposition is good enough. However, obtaining such an algorithm is non-trivial, since the fact that a subset of transitions is observable is global to the whole graph, not local.

Here, we consider simple components of an FSM $M$, that is, components which have only one entry and one exit point. Notice that simple components often occur in the control flow graphs generated by software programs, as control flow graphs of individual functions. For instance, $\left\{s_{1}, s_{2}, s_{3}, s_{4}\right\}$ form a simple component of the FSM in Fig. 2. Anyway, we will relax this condition in section 5.2. Formally, we call $C=\left(Q^{\prime}, s_{0}^{\prime}, s_{f}^{\prime}, \mathcal{T}^{\prime}\right)$ a simple component of $M=\left(Q, s_{0}, s_{f}, \mathcal{T}\right)$ when $Q^{\prime} \subsetneq Q, \mathcal{T}^{\prime} \subsetneq \mathcal{T}$, and
$\forall q \in Q \backslash Q^{\prime}, q^{\prime} \in Q^{\prime}$, we have $\left(q, q^{\prime}\right) \in \mathcal{T}$ or $\left(q^{\prime}, q\right) \in \mathcal{T}$ implies $q^{\prime} \in\left\{s_{0}^{\prime}, s_{f}^{\prime}\right\}$.

First, we need the following additional notations. Given $\mathcal{T}_{O}$, a path $\rho$ is said to be an unobserved path if it does not use any transitions of $\mathcal{T}_{O}$. For a service $M$ and a set of transitions $\mathcal{T}_{O}$ of $M$, we define the following predicates:

- $P_{0}\left(M, \mathcal{T}_{O}\right)$ holds if there does not exist more than one unobserved path between any two states $s_{1} \neq s_{2} \in Q$ ( $\mathcal{T}_{O}$ is an observable set of transitions).
- $P_{1}\left(M, \mathcal{T}_{O}\right)$ holds if (i) $P_{0}\left(M, \mathcal{T}_{O}\right)$ holds, and (ii) there does not exist an unobserved path from $s_{0}$ to $s_{f}$.
- $P_{1^{\prime}}\left(M, \mathcal{T}_{O}\right)$ holds if (i) $P_{0}\left(M, \mathcal{T}_{O}\right)$ holds, and (ii) there do not exist states $s_{1}, s_{2} \in Q$ such that (a) there is an unobserved path from $s_{0}$ to $s_{2}$, (b) there is an unobserved path from $s_{1}$ to $s_{f}$, and (c) there is an unobserved path from $s_{1}$ to $s_{2}$. We refer to such a combination of nodes and edges (e.g., Fig. 2) as an unobserved reverse cyclic pattern between $s_{1}$ and $s_{2}$ (within $M$ ).

By definition, $\quad P_{1^{\prime}}\left(M, \mathcal{T}_{O}\right) \Rightarrow P_{1}\left(M, \mathcal{T}_{O}\right) \Rightarrow$ $P_{0}\left(M, \mathcal{T}_{O}\right)$, since for all $s$, there always exists a path from $s$ to $s$. Let $\epsilon<0<1<1^{\prime}$. We define $\operatorname{Best}\left(M, \mathcal{T}_{O}\right)=x \in\left\{\epsilon, 0,1,1^{\prime}\right\}$ such that $P_{x}\left(M, \mathcal{T}_{O}\right)$ holds, but not $P_{x x}\left(M, \mathcal{T}_{O}\right)$ with $x x>x$, with the convention $P_{\epsilon}\left(M, \mathcal{T}_{O}\right)$ is always true. Informally, Best refers to the best properties a given set of transitions can ensure, if observed.

Proposition 2 Let $C$ be a simple component of $M$, and $\mathcal{T}_{1}, \mathcal{T}_{2}$ be subsets of transitions of $C$, respectively such that $\operatorname{Best}\left(C, \mathcal{T}_{1}\right)=\operatorname{Best}\left(C, \mathcal{T}_{2}\right)$. Then, for all subset of transitions $\mathcal{T}_{O}$ of $M \backslash C$, we have $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right)=$ $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{2}\right)$.

Proof. Let $C=\left(Q^{\prime}, s_{0}^{\prime}, s_{f}^{\prime}, \mathcal{T}^{\prime}\right)$, and $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup\right.$ $\left.\mathcal{T}_{1}\right)=x$. Let us assume that $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{2}\right)=\epsilon$, that is, there exists a pair of states $s_{1}, s_{2}$ of $M$ with unobserved (for $\mathcal{T}_{O} \cup \mathcal{T}_{2}$ ) paths $\rho_{1}, \rho_{2}$ from $s_{1}$ to $s_{2}$, such that the states traversed by $\rho_{1}$ and $\rho_{2}$ are disjoint, but for $s_{1}$ and $s_{2}$. We show now that $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right)=\epsilon$.

If both $\rho_{1}$ and $\rho_{2}$ do not touch $C$, then $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup\right.$ $\left.\mathcal{T}_{1}\right)=\epsilon$. If both $\rho_{1}$ and $\rho_{2}$ belong to $C$, then $P_{0}\left(C, \mathcal{T}_{2}\right)$ does not hold, which means $P_{0}\left(C, \mathcal{T}_{1}\right)$ does not hold, implying $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right)=\epsilon$.

If $s_{1}, s_{2} \in\left(Q \backslash Q^{\prime}\right) \cup\left\{s_{0}^{\prime}, s_{f}^{\prime}\right\}$, and $\rho=\rho_{1}$ or $\rho_{2}$ passes through $C$, then there exists an unobserved (for $\mathcal{T}_{2}$ ) path from $s_{0}^{\prime}$ to $s_{f}^{\prime}$ (a subpath of $\rho$ ). Given this, $P_{1}\left(C, \mathcal{T}_{2}\right)$ does not hold; which implies that $P_{1}\left(C, \mathcal{T}_{1}\right)$ does not hold. Hence, there exists an unobserved (for $\mathcal{T}_{1}$ ) path from $s_{0}^{\prime}$ to $s_{f}^{\prime}$, and an unobservable (for $\mathcal{T}_{O} \cup \mathcal{T}_{1}$ ) path $\rho^{\prime}$ can be constructed from this path and $\rho$. As such, there are two disjoint paths unobservable for $\mathcal{T}_{O} \cup \mathcal{T}_{1}$ between $s_{1}$ and $s_{2}$ : $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right)=\epsilon$.


(b)

Figure 4. Computation of (a) $F_{1}(M)$ and (b) $F_{1^{\prime}}(M)$.

If $s_{1}, s_{2} \in Q^{\prime}$, and $\rho=\rho_{1}$ or $\rho_{2}$ passes through $C$, then there exists an unobserved reverse cyclic pattern (for $\mathcal{T}_{2}$ ) between $s_{0}^{\prime}$ and $s_{f}^{\prime}$. Given this, $P_{1^{\prime}}\left(C, \mathcal{T}_{2}\right)$ does not hold; which implies that $P_{1^{\prime}}\left(C, \mathcal{T}_{1}\right)$ does not hold. Hence, there exists an unobserved (for $\mathcal{T}_{1}$ ) reverse cyclic pattern between $s_{0}^{\prime}$ and $s_{f}^{\prime}$, and an unobservable (for $\mathcal{T}_{O} \cup \mathcal{T}_{1}$ ) path $\rho^{\prime}$ can be constructed from this pattern and $\rho$. As such, there are two disjoint paths unobservable for $\mathcal{T}_{O} \cup \mathcal{T}_{1}$ between $s_{1}$ and $s_{2}$ : $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right)=\epsilon$.

The cases where $s_{1} \in Q^{\prime} \backslash\left\{s_{0}^{\prime}, s_{f}^{\prime}\right\}, s_{2} \notin Q^{\prime}$, or both $\rho_{1}, \rho_{2}$ pass through $C$, are not possible because then the paths would meet in $s_{0}^{\prime}$ and/or $s_{f}^{\prime}$.

Hence, $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{2}\right)=\epsilon \Longrightarrow \operatorname{Best}\left(M, \mathcal{T}_{O} \cup\right.$ $\left.\mathcal{T}_{1}\right)=\epsilon$. By symmetry between $\mathcal{T}_{1}$ and $\mathcal{T}_{2}$, we have the equivalence: $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{2}\right) \geq 0 \operatorname{iff} \operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right) \geq$ 0 . Now, for all $x \in\left\{1,1^{\prime}\right\}$, we can enrich $M$ to $F_{x}(M)$ with $\operatorname{Best}\left(M, \mathcal{T}_{O}\right) \geq x$ iff $\operatorname{Best}\left(F_{x}(M), \mathcal{T}_{O}\right) \geq 0$. Applying it to $M$, we get $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{2}\right)=\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right)$.

The functions $F_{x}$ are given schematically in Figure 4, where $F_{x}(M)=\left(Q^{\prime \prime}, s_{0}^{\prime \prime}, s_{f}^{\prime \prime}, \mathcal{T}^{\prime \prime}\right)$ :

For $x \in\left\{0,1,1^{\prime}\right\}$, we define $\mathcal{T}_{x}(M)$ as a smallest subset $\mathcal{T}_{O}$ of transitions of $M$ such that $P_{x}\left(M, \mathcal{T}_{O}\right)$ holds. For a subset of transitions $T$ of a component $C$ of $M$, we also denote by $\mathcal{T}_{x}^{T, C}(M)$ a smallest set $\mathcal{T}_{O}$ such that $\mathcal{T}_{O} \cap C=T$ and $P_{x}\left(M, \mathcal{T}_{O}\right)$ holds. As far as we know, every algorithm to compute the minimal observable set of transitions is recursive, taking the set of transitions considered observable as input. It is easy to modify them to input in the beginning not $\emptyset$ but $T$, and disallowing to select any new transitions in $C$, such that they compute $\mathcal{T}_{x}^{T, C}(M)$, and they do it faster than $\mathcal{T}_{x}(M)$ because they cannot choose among the transitions of $C$. As proved in proposition 2, the size of $\mathcal{T}_{O}$ is constant for several $T$ such that $\operatorname{Best}(C, T)=y$. If $\left|T^{\prime}\right|>|T|$ with $\operatorname{Best}(C, T)=\operatorname{Best}\left(C, T^{\prime}\right)$, then $\left|\mathcal{T}_{x}^{T^{\prime}, C}(M)\right|>\left|\mathcal{T}_{x}^{T, C}(M)\right|$. We can use this idea to compute $\mathcal{T}_{x}(M)$ in a compositional manner, for a service $M$ having simple component $C$ :

MinimalDecomposition $(M, C)$ :

1. Compute a minimal set $\mathcal{T}_{y}(C)$ of transitions of $C$, $\forall y \in\left\{0,1,1^{\prime}\right\}$.
2. Compute a minimal set $\mathcal{T}_{0}^{\mathcal{T}_{y}(C), C}(M)$ of transitions of $M$, for all y.


Figure 5. FSM $M=\left(Q, s_{0}, s_{f}, \mathcal{T}\right)$ having simple component $C=\left(Q^{\prime}, s_{0}^{\prime}, s_{f}^{\prime}, \mathcal{T}^{\prime}\right)$.
3. Output a set of smallest size among $\mathcal{T}_{0}^{\mathcal{T}_{y}(C), C}(M)$.

For example, consider the FSM $M$ having simple component $C$ in Fig. 5.

1. A minimal set $\mathcal{T}_{0}(C)=\left\{\left(s_{0}^{\prime}, s_{2}\right),\left(s_{1}, s_{f}^{\prime}\right)\right\}$, $\mathcal{T}_{1}(C)=\left\{\left(s_{0}^{\prime}, s_{1}\right),\left(s_{2}, s_{f}^{\prime}\right)\right\}$, and $\quad \mathcal{T}_{1^{\prime}}(C)=$ $\left\{\left(s_{0}^{\prime}, s_{2}\right),\left(s_{1}, s_{f}^{\prime}\right),\left(s_{1}, s_{2}\right)\right\}$.
2. The corresponding observable sets of $M$ : $\mathcal{T}_{0}^{\mathcal{T}_{0}(C), C}(M)=\left\{\left(s_{0}^{\prime}, s_{2}\right),\left(s_{1}, s_{f}^{\prime}\right), \quad\left(s_{0}, s_{f}\right)\right\}$ of size $3, \mathcal{T}_{0}^{\mathcal{T}_{1}(C), C}(M)=\left\{\left(s_{0}^{\prime}, s_{1}\right),\left(s_{2}, s_{f}^{\prime}\right)\right\}$ of size 2 , and $\mathcal{T}_{0}^{\mathcal{T}_{1^{\prime}}(C), C}(M)=\left\{\left(s_{0}^{\prime}, s_{2}\right),\left(s_{1}^{\prime}, s_{f}\right),\left(s_{1}, s_{2}\right)\right\}$ of size 3.
3. $\mathcal{T}_{0}^{\mathcal{T}_{1}(C), C}(M)$ is a minimal observable set of $M$.

Theorem 2 Given $C$ a simple component of $M$, MinimalDecomposition $(M, C)$ returns a minimal observable set of transitions of $M$ in time at most $O\left(2^{|C|}+2^{|M|-|C|}\right)$.

Proof. First, the brute force algorithm for computing $\mathcal{T}_{y}(C)=\mathcal{T}_{0}\left(F_{y}(C)\right)$ takes $O\left(2^{|C|}\right)$ time, and takes $O\left(2^{|M|-|C|}\right)$ for computing $\mathcal{T}_{0}^{\mathcal{T}_{y}(C), C}(M)$. Note that heuristics [8] or Sat solvers could be used instead of the brute force method.

For all $y \in\left\{0,1,1^{\prime}\right\}, \mathcal{T}_{0}^{\mathcal{T}_{y}(C), C}(M)$ is an observable set of transitions by definition, but not necessarily minimal. Let $\mathcal{T}=\mathcal{T}_{O} \cup \mathcal{T}_{1}$ be a minimal observable set of transitions with $\mathcal{T}_{1}$ the transitions of $\mathcal{T}$ in $C$ and $\mathcal{T}_{O}=\mathcal{T} \backslash \mathcal{T}_{1}$. Let $y=$ $\operatorname{Best}\left(C, \mathcal{T}_{1}\right)$. Then, $\left.\left|\mathcal{T}_{y}(C)\right| \leq \mid \mathcal{T}_{1}\right) \mid$. Moreover, applying proposition 2, we get $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right)=\operatorname{Best}\left(M, \mathcal{T}_{O} \cup\right.$ $\left.\mathcal{T}_{y}(C)\right) \geq 0$, that is, $\left|\mathcal{T}_{0}^{\mathcal{T}_{y}(C), C}(M)\right| \leq\left|\mathcal{T}_{O}\right|+\left|\mathcal{T}_{y}(C)\right| \leq$ $\left|\mathcal{T}_{O}\right|+\left|\mathcal{T}_{1}\right|=|\mathcal{T}|$. By definition of $\mathcal{T}_{0}^{\mathcal{T}_{y}(C), C}(M)$, it is an observable set of transitions, and by minimality of $\mathcal{T}$, we get $\left|\mathcal{T}_{0}^{\mathcal{T}_{y}(C), C}(M)\right|=|\mathcal{T}|$. Hence, one of the sets computed by MinimalDecomposition is a minimal observable set.

Finally, it remains to explain how to find simple components of an FSM. We present here a linear time (in the number of transitions) algorithm to compute the smallest simple components $C$ of an FSM $M$ knowing its starting and ending state, and an initial transition from starting state.

Input. A service $M=\left(Q, s_{0}, s_{f}, \mathcal{T}\right), \tau=\left(s, s_{1}\right) \in \mathcal{T}$ and $t \in Q$.

Output. The smallest simple component $C=$ ( $\left.Q^{\prime}, s, t, \mathcal{T}^{\prime}\right)$ of $M$ with $\tau \in \mathcal{T}^{\prime}$.

Initialization. $\mathcal{T}^{\prime}=\phi, S=\{\tau\}, Q^{\prime}=\left\{s, s_{1}, t\right\}$.

1. Select a transition $\tau^{\prime}=\left(s_{1}^{\prime}, s_{2}^{\prime}\right) \in S$. If $s_{2}^{\prime} \neq t$, then $S=S \cup\left\{\left(s_{2}^{\prime}, s_{3}^{\prime}\right) \mid\left(s_{2}^{\prime}, s_{3}^{\prime}\right) \in \mathcal{T} \backslash \mathcal{T}^{\prime}\right\}$. If $s_{1}^{\prime} \neq s$, then $S=S \cup\left\{\left(s_{3}^{\prime}, s_{1}^{\prime}\right) \mid\left(s_{3}^{\prime}, s_{1}^{\prime}\right) \in \mathcal{T} \backslash \mathcal{T}^{\prime}\right\}$. Finally, $S=S \backslash\left\{\tau^{\prime}\right\}, \mathcal{T}^{\prime}=\mathcal{T}^{\prime} \cup\left\{\tau^{\prime}\right\}$, and $Q^{\prime}=Q^{\prime} \cup\left\{s_{1}^{\prime}, s_{2}^{\prime}\right\}$.
2. If $S \neq \emptyset$, repeat step 1 .
3. If $\left(s \neq s_{0} \wedge s_{0} \in Q^{\prime}\right) \vee\left(t \neq s_{f} \wedge s_{f} \in Q^{\prime}\right)$, then return that a simple component between $s$ and $t$ with respect to $\tau$ does not exist. Else, return $C$.

The above algorithm can be iteratively invoked to compute the set $S_{C}$ of all simple components of an FSM $M$. We now give an algorithm to compute the largest simple component of $M$.

1. For a pair of components $D, E \in S_{C}$, if $D$ is a subgraph of $E$, delete $D$.
2. For a pair of components $D=\left(Q^{\prime}, s_{0}^{\prime}, s_{f}^{\prime}, \mathcal{T}^{\prime}\right)$ and $E=\left(Q^{\prime \prime}, s_{0}^{\prime \prime}, s_{f}^{\prime \prime}, \mathcal{T}^{\prime \prime}\right)$ of $S_{C}$, if $s_{0}^{\prime}=s_{0}^{\prime \prime}$ and $s_{f}^{\prime}=s_{f}^{\prime \prime}$, then create a new component $F=\left(Q^{\prime} \cup Q^{\prime \prime}, s_{0}^{\prime}, s_{f}^{\prime}\right.$, $\left.\mathcal{T}^{\prime} \cup \mathcal{T}^{\prime \prime}\right)$. If $F \neq M$, then delete $D$ and $E$ from $S_{C}$, and add $F$ to $S_{C}$.
3. Return the biggest $C \in S_{C}$.

### 4.1 Experimental Results

We tested our decomposition algorithm on acyclic graphs randomly generated, using the Synthetic DAG generation tool (http://www.loria.fr/~suter/dags.html), with 512MB heap memory. Usually, the graphs randomly generated look very unnatural, and as expected, there are really few simple components in the graph. Moreover, very big components (with about $85 \%$ of edges) remains non decomposable. Our algorithm finds these components within seconds, which allows a speed up of about four times. Unfortunately, it does not change the exponential scaling. The algorithm runs out of memory at around 23 edges (instead of around 20 edges without decomposition).

Random graphs generated using the parameter "-regular $x "(x \leq 0.2)$, that is the distribution of tasks between the different levels is fairly regular, look more natural. For them, our algorithm finds non trivial components. For instance, for a system with 31 edges, the biggest undecomposable component has only 18 edges. The algorithm finds an observable set of 7 transitions in 3 seconds when decomposition is used, instead of 4 minutes. Without the decomposition method, the algorithm runs out of memory at about 21
edges, while it runs out of memory at about 33 edges with the decomposition algorithm. Using an optimized algorithm instead of brute force would allow a much better scaling in both cases.

We found several workflows in the literature, that we automatically transformed into graphs. For instance, we modeled the workflow from [5] using 12 nodes and 26 edges (see http://www.crans.org/rgenest/dns/). Without the decomposition method, the brute force method goes out of memory. Our algorithm decomposes the graph within one second. Component $C_{1}$ is found. In component $C_{1}$, component $C_{2}$ is found and so on until component $C_{11}$. Our algorithm then uses the brute force method on $C_{11}$ and $C_{i} \backslash C_{i-1}$ (we denote by $C_{0}$ the original graph). Since $C_{11}$ has 1 edge, and $C_{i}-C_{i-1}$ has 3 edges for $3 \leq i \leq 9$ and else 1 edge, the brute force algorithm was very efficient, finishing also within one second. We found that the minimal observable set of transitions has 15 edges. Moreover, logging every action of this workflow takes around 1 MB , while it takes 200 KB to log a minimal observable set of transitions.

## 5 More General Services

We overcome the limitations of the previous method in this section, namely by allowing several components, with more than one entry and exit state, and distributed systems.

### 5.1 Hierarchical Services

The decomposition into a simple component can be extended with the component being in turn decomposed and so on. Even better, if a component is used several times (as is the case for large composite services), it does not need to be recomputed several times. In order to formalize such a framework, we turn to hierarchical services.

Hierarchical services provide an efficient way to model large and complex services by allowing a modular decomposition of the problem space. In particular, we consider hierarchical services where two transitions (supertransitions) can be further refined into an FSM. A hierarchical FSM $H$ is a finite sequence $\left\langle M_{i}\right\rangle_{i=1 \cdots n}$, where $M^{i}=\left(Q^{i}, s_{0}^{i}, s_{f}^{i}, \mathcal{T}^{i},\left(\tau_{1}^{i}, k_{1}^{i}\right),\left(\tau_{2}^{i}, k_{2}^{i}\right)\right)$ is defined as follows:

- $Q^{i}$ is the finite set of states,
- $s_{0}^{i}$ and $s_{f}^{i}$ are the initial and final states, respectively,
- $\mathcal{T}^{i} \subseteq Q^{i} \times Q^{i}$ are the transitions,
- $\tau_{1}^{i}, \tau_{2}^{i} \in \mathcal{T}^{i} \cup\{\epsilon\}$ are two supertransitions representing services $M^{k_{1}^{i}}, M^{k_{2}^{i}}$ respectively, with $k_{1}^{i}, k_{2}^{i}>i$.

With each hierarchical FSM $H$, we associate an ordinary FSM $\mathcal{H}$ obtained by taking $M^{i}$, and recursively substituting


Figure 6. Hierarchical representation of programs with no goto statements
each supertransition by the FSM it represents. We define the size of a hierarchical FSM $H$ as the sum of the number of transitions of its components $M^{i}$. Its diameter is the number of transitions of $\mathcal{H}$. The size $H$ can be logarithmic in the diameter of $H$. Applying dynamic programing with Theroem 2 as base case leads to the following result:
Theorem 3 Let $H=\left(M_{i}\right)_{i=1}^{n}$ be a hierarchical FSM. It is NP-complete in the size of $H$ to compute $M O(\mathcal{H})$. Moreover, it takes at most time $O\left(\sum_{i=1}^{n} 2^{\left|M_{i}\right|}\right)$.

Proof. It suffices to compute a minimal set $\mathcal{T}_{v}\left(M_{i}\right)$ of transitions of $M_{i}$, for all $v \in V=\left\{0,1,1^{\prime}\right\}$. These actions are performed from bottom of the hierarchy to top. To compute $\mathcal{T}_{v}\left(C_{i}\right)$ for a module $M_{i}$ using $M_{j}, M_{k}, j, k>i$, we use $\mathcal{T}_{v^{\prime}}\left(M_{j}\right)$ and $\mathcal{T}_{v^{\prime \prime}}\left(M_{k}\right)$ which have already been computed. Indeed, it suffices to compute a minimal set $\mathcal{T}_{v}^{\mathcal{T}_{v^{\prime}}\left(M_{i}\right) \cup \mathcal{T}_{v^{\prime \prime}}\left(M_{k}\right), M_{j} \cup M_{k}}(M)$ of transitions of $M$, for all valuations $v^{\prime}, v^{\prime \prime}$. Then, it suffices to output a set among $\mathcal{T}_{v}^{\mathcal{T}_{v^{\prime}}\left(M_{i}\right) \cup \mathcal{T}_{v^{\prime \prime}}\left(M_{k}\right), M_{j} \cup M_{k}}(M)$ of minimal size.

The best case comparison is with respect to a hierarchical service of diameter $O\left(2^{n}\right)$, using $n$ components of size 2 . The brute force non-compositional method run on $\mathcal{H}$ takes time $O\left(2^{2^{n}}\right)$, while our method takes $O(n)$, that is a doubly exponential improvement (one exponential due to the reuse of components, and another due to decomposition).

In addition to reducing the problem space, another motivation for considering hierarchical systems is to study systems having desirable properties. For instance, let us consider programs where goto statements (or interleaving branches) are not allowed. Such a hierarchical system representation $H=\left\langle M_{1}, C\right\rangle$ is given in Fig. 6. Then, $M O(\mathcal{H})=n \times M O(C)$, where $n$ is the number of times $C$ occurs in $\mathcal{H}$. Basically, for $C=\left(Q^{\prime}, s_{0}^{\prime}, s_{f}^{\prime}, \mathcal{T}^{\prime}\right)$, if there exists $m$ paths (with no interleaving amongst them) from $s_{0}^{\prime}$ to $s_{f}^{\prime}$, then $M O(C)=m-1$. Given this, to compute $M O(\mathcal{H})$, for each occurrence of $C$ in $\mathcal{H}=\left(Q, s_{0}, s_{f}, \mathcal{T}\right)$; we can collapse $C$ and make a recursive call to compute $M O\left(\left(Q \backslash Q^{\prime} \cup\left\{s_{0}^{\prime}, s_{f}^{\prime}\right\}\right)\right.$. The structure guarantees that at the termination of such a recursive invocation, $\mathcal{H}=C$.

### 5.2 Complex Decompositions

The previous results are quite encouraging, but not totally satisfactory as simple decompositions may not be enough to reduce the size of the components. Indeed, using components with several entry and exit states (complex decomposition) help reducing further the size of the components.

Assume that a service $M=\left(Q, s_{0_{1}}, \cdots, s_{0_{b_{1}}}\right.$, $\left.s_{f_{1}}, \cdots, s_{f_{b_{2}}}, \mathcal{T}\right)$ has a set $P$ of $b=b_{1}+b_{2}$ port states, consisting of $b_{1}$ input states $\left(s_{0}^{i}\right)_{i=1}^{b_{1}}$ and $b_{2}$ output states $\left(s_{f}^{i}\right)_{i=1}^{b_{2}}$. Let $\mathcal{T}_{O}$ be a set of transitions of $S$. We define $b^{2}+1$ predicates $P_{0}\left(M, \mathcal{T}_{O}\right), P_{p_{1}, p_{2}}\left(M, \mathcal{T}_{O}\right)$ for all $p_{1}, p_{2} \in P$.

- $P_{0}\left(M, \mathcal{T}_{O}\right)$ holds if there does not exist more than one unobserved path between any two states $s_{1} \neq s_{2} \in Q$ ( $\mathcal{T}_{O}$ is an observable set of transitions).
- if $p_{1}$ is an input and $p_{2}$ an output, $P_{p_{1}, p_{2}}\left(M, \mathcal{T}_{O}\right)$ holds if (i) $P_{0}\left(M, \mathcal{T}_{O}\right)$ holds, and (ii) there does not exist an unobserved path from $p_{1}$ to $p_{2}$.
- if $p_{1}$ is an output and $p_{2}$ an input, $P_{p_{1}, p_{2}}\left(M, \mathcal{T}_{O}\right)$ holds if (i) $P_{0}\left(M, \mathcal{T}_{O}\right)$ holds, and (ii) there do not exist states $s_{1}, s_{2} \in Q$ such that (a) there is an unobserved path from $p_{2}$ to $s_{2}$, (b) there is an unobserved path from $s_{1}$ to $p_{1}$, and (c) there is an unobserved path from $s_{1}$ to $s_{2}$.
- if $p_{1}, p_{2}$ are two inputs, $P_{p_{1}, p_{2}}\left(M, \mathcal{T}_{O}\right)$ holds if (i) $P_{0}\left(M, \mathcal{T}_{O}\right)$ holds, and (ii) there does not exist a state $s$ in $M$ with unobserved paths from both $p_{1}$ and $p_{2}$ to $s$.
- if $p_{1}, p_{2}$ are two outputs, $P_{p_{1}, p_{2}}\left(M, \mathcal{T}_{O}\right)$ holds if (i) $P_{0}\left(M, \mathcal{T}_{O}\right)$ holds, and (ii) there does not exist a state $s$ in $M$ with unobserved paths from $s$ to both $p_{1}$ and $p_{2}$.

Notice that some predicates imply others, as $P_{p_{1}, p_{2}}=$ $P_{p_{2}, p_{1}}$ for two inputs or outputs, and $P_{p_{1}, p_{2}} \Longrightarrow P_{p_{2}, p_{1}}$ for $p_{1}$ output and $p_{2}$ input. However, we do not have a total order between predicates. That is, we define $\operatorname{Best}\left(M, \mathcal{T}_{O}\right)$ : $P^{2} \rightarrow\{0,1\}$ as a function with $\operatorname{Best}\left(M, \mathcal{T}_{O}\right)\left(p_{1}, p_{2}\right)=1$ iff $P_{p_{1}, p_{2}}\left(M, \mathcal{T}_{O}\right)$. We can then extend the proof of proposition 2 to obtain the following porposition (proof omitted because of lack of space):

Proposition 3 Let $C$ be a component of $M$, and $\mathcal{T}_{1}, \mathcal{T}_{2}$ be subsets of transitions of $C$, respectively such that $\operatorname{Best}\left(C, \mathcal{T}_{1}\right)=\operatorname{Best}\left(C, \mathcal{T}_{2}\right)$. Then, for all subset of transitions $\mathcal{T}_{O}$ of $M \backslash C$, we have $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{1}\right)=$ $\operatorname{Best}\left(M, \mathcal{T}_{O} \cup \mathcal{T}_{2}\right)$.

We can use the above theorem to compute a minimal observable set for an FSM $M$ having component $C$ as follows:

MinimalDecomposition-Complex ( $M, C$ ):

1. Compute a minimal set $\mathcal{T}_{v}(C)$ of transitions of $C$, for all valuation $v: P^{2} \rightarrow\{0,1\}$.
2. Compute a minimal set $\mathcal{T}_{0}^{\mathcal{T}_{v}(C), C}(M)$ of transitions of $M$, for all v .
3. Output a set of smallest size among $\mathcal{T}_{0}^{\mathcal{T}_{v}(C), C}(M)$.

It allows to extend the previous result on simple components, for instance for hierarchical services, where each component uses at most $p$ ports:

Theorem 4 Let $p \geq 1$. It is NP-complete in the size of a hierarchical service $H=\left(M_{i}\right)_{i=1}^{n}$ using at most $p$ ports to compute $M O(\mathcal{H})$, with $p$ fixed. Moreover, it takes time at most $O\left(2^{3 p^{2}} \sum_{i=1}^{n} 2^{\left|M_{i}\right|}\right)$.

Proof. Let us consider a hierarchical system $\left(C_{i}\right)_{i=1, \cdots, n}$. The time complexity comes directly from Proposition 3 and the same dynamic programing algorithm as Theorem 3. It suffices to compute a minimal set $\mathcal{T}_{v}\left(C_{i}\right)$ of transitions of $C_{i}$, for all valuation $v: P^{2} \rightarrow$ $\{0,1\}$. We call $V$ the set of valuations. These actions are performed from bottom of the hierarchy to top. To compute $\mathcal{T}_{v}\left(C_{i}\right)$ for a module $C_{i}$ using $C_{j}, C_{k}, j, k>i$, we use $\mathcal{T}_{v^{\prime}}\left(C_{j}\right)$ and $\mathcal{T}_{v^{\prime \prime}}\left(C_{k}\right)$ which have already been computed. Indeed, it suffices to compute a minimal set $\mathcal{T}_{v}^{\mathcal{T}_{v^{\prime}}\left(C_{i}\right) \cup \mathcal{T}_{v^{\prime \prime}}\left(C_{k}\right), C_{j} \cup C_{k}}(M)$ of transitions of $M$, for all valuations $v^{\prime}, v^{\prime \prime}$. Then, it suffices to output a set of minimal size among the $\mathcal{T}_{v}^{\mathcal{T}_{v^{\prime}}\left(C_{i}\right) \cup \mathcal{T}_{v^{\prime \prime}}\left(C_{k}\right), C_{j} \cup C_{k}}(M)$ computed.

Let us turn now to the NP-complete proof. The NP-hard part follows from the NP-hardness in the non hierarchical case. We give a proof that can be checked in polynomial time, which proves the NP inclusion.

A proof that $M O(\mathcal{H}) \leq k$ is given by $n 2^{p^{2}}$ data $f(i, v)=\left(T, v^{\prime}, v^{\prime \prime}\right)$, where $T$ is a subset of transitions of $C_{i}$, and $v^{\prime}, v^{\prime \prime}$ two valuations. The proof is correct if we have:

- $\operatorname{size}(1, v=0) \leq k$, where size is defined inductively as $\operatorname{size}(i, v)=|T|+\operatorname{size}\left(j, v^{\prime}\right)+\operatorname{size}\left(k, v^{\prime \prime}\right)$, where $f(i, v)=\left(T, v^{\prime}, v^{\prime \prime}\right)$ and $C_{i}$ uses components $C_{j}$ and $C_{k}$,
- for all $i, v$, if $f(i, v)=\left(T, v^{\prime}, v^{\prime \prime}\right)$, then deleting from $C_{i}$ transitions of $T$, the resulting graph has property $P_{v}$ assuming that the two supertransitions have properties $P_{v^{\prime}}$ and $P_{v^{\prime \prime}}$ respectively. We say that a graph have properties $P_{v}$ when $P_{p, q}$ is true whenever $v(p, q)=1$.

Indeed, if we have such a proof, then we can choose an observable alphabet for $\left(C_{n}, v\right)$ of size $\operatorname{size}(n, v)$, and so on until an alphabet of $\operatorname{size}(1, v=0)$. The correctness of such a proof can be checked in Polynomial time.

Assume that $M O(\mathcal{H}) \leq k$. It remains to prove that there exists such a proof. Let $\left|\mathcal{T}_{O}\right| \leq k$ be an observable alphabet
of $\mathcal{H}$. Then on each component $C$ of $\mathcal{H}$ which corresponds to some $H_{i}$, we can compute $\operatorname{Best}\left(C,\left.\mathcal{T}_{O}\right|_{C}\right)=v$. We do the same with its subcomponent $D, E, \operatorname{Best}\left(D,\left.\mathcal{T}_{O}\right|_{D}\right)=v^{\prime}$ $\operatorname{Best}\left(E,\left.\mathcal{T}_{O}\right|_{E}\right)=v^{\prime \prime}$. Then we input in the proof $f(i, v)=$ $\left(\left.\mathcal{T}_{O}\right|_{C \backslash(D \cup E)}, v^{\prime}, v^{\prime \prime}\right)$.

The problem is that there are many components $C$ corresponding to $H_{i}$, and several can have the same best properties $v$. We fill the entry with some component $C$ having the smallest $\left.\mathcal{T}_{O}\right|_{C}$. The reason why we can do it is given by proposition 3, that is we can replace any set of transitions by another one, provided we keep the same best properties. Taking the smallest ensure that the proof will show an alphabet of size at most $\left|\mathcal{T}_{O}\right|$.

If there are some empty inputs, e.g., component $H_{i}$ for which we never find $\operatorname{Best}\left(H_{i},\left.\mathcal{T}_{O}\right|_{H_{i}}\right)=(x, y, z)$, then it is irrelevant, and hence useless in the proof.

A simple rule of thumb to know whether it is worth applying our technique on a component is when it has a high number of transitions with respect to the square of the number of its ports. In order to find components having few ports, one can use heuristics on graph partitioning [9].

### 5.3 Distributed Services

We consider distributed services, given in the form of product of two services $M$ and $N$. We first on services having no interaction between them (that is, one service cannot write a global variable that is read or written by another service). We explain later how to deal with interacting services having a non interacting component.

The composition schema of such a composite service is specified as a product of the FSM's (corresponding to the composition schema) of the component composite services [3, 6]. Given services $M=\left(Q_{1}, s_{0_{1}}, s_{f_{1}}, \mathcal{T}_{1}\right)$ and $N=$ $\left(Q_{2}, s_{0_{2}}, s_{f_{2}}, \mathcal{T}_{2}\right), \mathcal{T}_{1} \cap \mathcal{T}_{2}=\emptyset$, we define their product $M \times N=\left(Q, s_{0_{1}} s_{0_{2}}, s_{f_{1}} s_{f_{2}}, \mathcal{T}\right)$ with $Q=Q_{1} \times Q_{2}$ and

$$
\mathcal{T}=\mathcal{T}_{1} \times \mathcal{T}_{2 \epsilon} \cup \mathcal{T}_{2} \times \mathcal{T}_{1 \epsilon}
$$

where $\mathcal{T}_{k \epsilon}$ is the set of self loop transitions $\left(s_{i}, s_{i}\right)$ for $s_{i} \in Q_{k}, k=1,2$.

The observation and logging for each component service are done locally. Hence, our decomposition method cannot be applied on the product since choosing to observe a transition in a component of the product might force it to be observed in another as well (if the same component is reused). Moreover, strict execution sequence detection is not required, since not knowing the exact interleaving between two "equivalent paths" is not needed. More formally, two consecutive transitions $\tau_{i}$ and $\tau_{i+1}$ of a path $\rho$ of $M \times N$ can commute if $\tau_{i}$ is from $M$ and $\tau_{i+1}$ from $N$, or vice versa. Two paths $\rho_{1} \neq \rho_{2}$ of $M \times N$ are equivalent $\rho_{1} \equiv \rho_{2}$, if $\rho_{1}=\rho_{2}$ after a finite sequence of commutations on $\rho_{1}$ (or $\rho_{2}$ ). We say that the product $M \times N$ is


Figure 7. FSM's (a) $M$ (b) $N$ (c) $M \times N$ (dashed arrows represent observable transitions).
observation sequence detectable with the observation of $\mathcal{T}_{1}$ in $M$ and $\mathcal{T}_{2}$ in $N$ if for all paths $\rho_{1}, \rho_{2}$ of $M \times N$ such that $\operatorname{Obs}_{\mathcal{T}_{1} \cup \mathcal{T}_{2}}^{\text {last }}\left(\rho_{1}\right)=\operatorname{Obs}_{\mathcal{T}_{1} \cup \mathcal{T}_{2}}^{\text {last }}\left(\rho_{2}\right)$, we have $\rho_{1} \equiv \rho_{2}$. In this case, compensation can be performed using any of the equivalent (reversed) runs, this will result in the same consistent state. We then have the following desirable property:

Proposition 4 For a pair of non interacting FSM's $M$ and $N$, and one of their respective minimal observable sets $\mathcal{T}_{O M}$ and $\mathcal{T}_{O N}, \mathcal{T}_{O M} \cup \mathcal{T}_{O N}$ is a minimal observable set of $M \times N$.

Now, let us consider the more general case where $M$ and $N$ interact, but $M$ has a component $C$ that does not interact with $N$. Then, one can decompose $M$ into $C$ and $M \backslash C$, and compute an observable set $T_{i}$ of transitions of $C$. We can choose the observations of $C \times \mathcal{T}_{2 \epsilon}$ to be one of the $T_{i}$.

Example 2 We consider FSM's $M=\left(Q_{1}, s_{0_{1}}, s_{f_{1}}, \mathcal{T}_{1}\right)$ (Fig. 7a) and $N=\left(Q_{2}, s_{0_{2}}, s_{f_{2}}, \mathcal{T}_{2}\right)$ (Fig. 7b) representing e-services which allow searching and listening to songs online [3]. The e-services allow different modes of payment and searching for song files by singer/title. Their product $M \times N$ is shown in Fig. 7c. The FSM $M$ contains a simple component $C=\left(Q^{\prime}, s_{0_{1}}, s_{1}, \mathcal{T}^{\prime}\right)$ such that $\mathcal{T}^{\prime} \cap \mathcal{T}_{2}=\emptyset$. Now, let us consider minimal observable sets $\mathcal{T}_{O_{1}}=\{a\}$ and $\mathcal{T}_{O_{2}}=\{e\}$ of $C$ and $M \backslash C \times N$, respectively. Then, $\mathcal{T}_{O_{1}} \cup \mathcal{T}_{O_{2}}$ is a minimal observable set of $M \times N$ as shown in Fig. 7c (with the dashed arrows representing observable transitions).

## 6 Conclusion

We studied compensation under partial log visibility. To the best of our knowledge, this problem has never been considered in the context of transactional services. With respect to (federated) multi-databases, the problem is analogous to designing a global concurrency control protocol in the absence of complete information of the conflicts at different sites [7]. Here, we take the alternate approach and try to determine the absolute minimal set of actions that needs to be logged such that the service is always compensable (execution sequence detectable). We give a general divide and conquer framework which works on complex hierarchical distributed systems, and gives the absolute minimum number of transition to observe in order to get observability. It provides good complexity results (up to two exponentials better than the brute force method). As future work, we are investigating fast algorithms which give an approximated size of the minimal observable set of transitions. That is, they give an observable set of transitions, but it may not be minimal. Notice that our decomposition algorithm could be used to improve accuracy (decrease the size) in this approximated framework.

## References

[1] G. Alonso, F. Casati, H. Kuno, and V. Machiraju. Web Services: Concepts, Architecture and Applications. Springer Verlag 2004, ISBN: 3540440089.
[2] R. Alur, and M. Yannakakis. Model checking of hierarchical state machines. ACM TOPLAS 23(3):1-31, 2001.
[3] D. Berardi, D. Calvanese, G. Giacomo, M. Lenzerini, and M. Mecella Automatic Composition of e-Services that Export their Behavior. In ICSOC'04:621-630.
[4] D. Biswas. Compensation in the World of Web Services Composition. In SWSWPC'04, LNCS 3387:6980.
[5] W. Emmerich , B. Butchart, L. Chen, B. Wassermann and S. Price. Grid Service Orchestration Using the Business Process Execution Language (BPEL). Journal of Grid Computing3(3-4):283-304, 2005.
[6] X. Fu, T. Bultan, and J. Su. Analysis of Interacting BPEL Web Services. In $W W W W^{\prime} 04: 621-630$.
[7] D. Georgakopoulos, M. Rusinkiewicz, and A. Sheth. Using Tickets to Enforce the Serializability of Multidatabase Transactions. IEEE TKDE 6(1): 166-180, 1994.
[8] A. Haji-Valizadeh, and K. Loparo Minimizing the cardinality of an events set for supervisors of discreteevent dynamical systems. IEEE Transactions on Automatic Control, 41:1579-1593, 1996.
[9] B.W. Kernighan, and S. Lin. An Efficient Heuristic Procedure for Partitioning Graphs. The Bell Sys. Tech. Journal, 49(2): 291-307, 1970.
[10] R. Kumar, and V. Garg. Modeling and Control of Logical Discrete Event Systems. Kluwer, 1995.
[11] F. Lin, and W. Wonham. On observability of discreteevent systems. Information Sciences, 44(3):173-198, 1988.
[12] M. Lohrey, and S. Manneth. The complexity of tree automata and XPath on grammar-compressed trees. Theoretical Computer Science 363(2):196-210, 2006.
[13] S. Maheshwari. Traversal Marker Placement Problems Are NP-Complete. Research Report http://www.cs.colorado.edu/department/publications/ reports/docs/CU-CS-092-76.pdf. Boulder Univ.
[14] C. Ozveren, and A. Wilsky. Observability of Discrete Event Dynamical Systems. IEEE Transactions on Automatic Control, 35(7):797-806, 1990.
[15] W. Plandowski, and W. Rytter. Complexity of language recognition problems for compressed words. In Jewels are Forever:262-272, Springer, 1999.
[16] K. Rohloff, and J. van Schuppen. Approximating the minimal cost sensor selection for discrete-event systems. JDEDS 16(1):143-170, 2006.
[17] W. Sadiq, and M. Orlowska. Analyzing Process Models Using Graph Reduction Techniques. Inf. Syst. 25(2):117-134, 2000.
[18] M. Sampath, R. Sengupta, S. Lafortune, K. Sinaamohideen, and D. Teneketzis. Diagnosability of discrete event systems. IEEE Transactions on Automatic Control, 40(9):1555-1575, September 1995.
[19] A. Wombacher, P. Fankhauser, and E. Neuhold. Transforming BPEL into Annotated Deterministic Finite State Automata for Service Discovery. In IEEE ICWS'04:316-323.
[20] T. Yoo, and S. Lafortune. NP-completeness of sensor selection problems arising in partially observed discrete-event systems. IEEE Transactions on Automatic Control, 35(7):797-806, 1990.


[^0]:    *This work is supported by la Region Bretagne (CREATE ACTIVEDOC) and ANR-06-MDCA-005 DOCFLOW.

[^1]:    ${ }^{1}$ Notice that we could deal with a service without these requirements, but the proof would be more technical.

