

$$H(k) = - \sum_j [e^{ik} |j\rangle \langle j+1| + e^{-ik} |j+1\rangle \langle j|]$$

Translation

la maille élémentaire (grande) est (\vec{a}, \vec{b})

$$\hat{H}_0 = \sum_{\langle i,j \rangle} |i\rangle \langle j|, \text{ liaisons fortes } t \equiv 1$$

$$\hat{H} = \sum_{\langle i,j \rangle} e^{i2\pi \int_i^j d\vec{e} \cdot \vec{A}(\vec{r})} |i\rangle \langle j|, \phi_0 \equiv 1$$

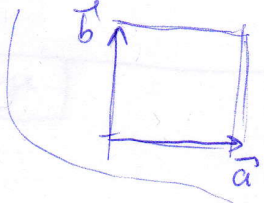
$$\hat{H} |\psi\rangle = E |\psi\rangle$$

$$\langle \vec{r} | \hat{T}_{\vec{a}} |\psi\rangle = \langle \vec{r} - \vec{a} | \psi \rangle = \psi(\vec{r} - \vec{a})$$

translation simple

$$\hat{T}_{\vec{a}} \hat{H} (\hat{T}_{\vec{a}})^{-1} \hat{T}_{\vec{a}} |\psi\rangle = E \hat{T}_{\vec{a}} |\psi\rangle \quad (\hat{T}_{\vec{a}})^{-1} = \hat{T}_{\vec{a}}^{\dagger} = \hat{T}_{-\vec{a}}$$

$T_a |x\rangle = |x+a\rangle$
 $\langle x | T_a = \langle x-a |$
 $\langle x | T_a |\psi\rangle = \langle x-a | \psi \rangle = \psi(x-a)$
 $T_a^{-1} = T_{-a} = T_a^{\dagger}$
 $T_{-a} |x\rangle = |x-a\rangle$
 $\langle x | T_{-a} = \langle x+a |$



$$\hookrightarrow \sum_{\langle i,j \rangle} e^{i2\pi \int_i^j d\vec{e} \cdot \vec{A}(\vec{r})} |i+\vec{a}\rangle \langle j+\vec{a}|$$

$$= \sum_{\langle i,j \rangle} e^{i2\pi \int_{i+\vec{a}}^{j+\vec{a}} d\vec{r} \cdot \vec{A}(\vec{r})} |i\rangle \langle j|$$

$$\int_i^j d\vec{r} \cdot \vec{A}(\vec{r}) \xrightarrow{-\vec{a}}$$

$$\begin{aligned} i' &= i + \vec{a} & i &= i' - \vec{a} \\ j' &= j + \vec{a} & j &= j' - \vec{a} \end{aligned}$$

~~$$\vec{r} = \vec{r}' - \vec{a}$$~~

$$= \sum_{\langle i,j \rangle} e^{i2\pi \int_i^j d\vec{r} \cdot \vec{A}(\vec{a} + \vec{r})} |i\rangle \langle j| \neq \hat{H}$$

donc $[\hat{T}_{\vec{a}}, \hat{H}] \neq 0$

il faut faire une translation de jauge

Translation de jauge

$$\hat{H} |\psi\rangle = E |\psi\rangle$$

~~$$e^{i2\pi \chi(\vec{r})} \hat{H} e^{-i2\pi \chi(\vec{r})} e^{i2\pi \chi(\vec{r})} |\psi\rangle = E e^{i2\pi \chi(\vec{r})} |\psi\rangle$$~~

$$\underbrace{e^{i2\pi \chi(\vec{r})}}_{|\tilde{\psi}\rangle} \hat{H} \underbrace{e^{-i2\pi \chi(\vec{r})}}_{|\tilde{\psi}\rangle} |\psi\rangle = E \underbrace{e^{i2\pi \chi(\vec{r})}}_{|\tilde{\psi}\rangle} |\psi\rangle$$

$$e^{i2\pi\chi(\vec{r})} \hat{H} e^{-i2\pi\chi(\vec{r})}$$

$$= \sum_{\langle i|j\rangle} e^{i2\pi\chi(\vec{r}_i)} e^{-i2\pi\chi(\vec{r}_j)} e^{i2\pi \int_i^j d\vec{r} \cdot \vec{A}(\vec{r})} |i\rangle \langle j|$$

Translation magnétique = translation + transfo de jauge

$$e^{i2\pi\chi(\vec{r})} \hat{T}_{\vec{a}} \hat{H} \hat{T}_{-\vec{a}} e^{-i2\pi\chi(\vec{r})}$$

$$= \sum_{\langle i|j\rangle} e^{i2\pi \left[\chi(\vec{r}_i) - \chi(\vec{r}_j) + \int_i^j d\vec{r} \cdot \vec{A}(\vec{a}+\vec{r}) \right]} |i\rangle \langle j| = \sum_{\langle i|j\rangle} e^{i2\pi \int_i^j d\vec{r} \cdot \vec{A}(\vec{r})} |i\rangle \langle j|$$

on choisit $\chi(\vec{r})$ tq $\chi = \int_0^{\vec{r}} d\vec{r}' \cdot \vec{A}(\vec{r}')$ pour que i.e.

$$\chi_{\vec{a}}(\vec{r}_i) - \chi_{\vec{a}}(\vec{r}_j) = \int_i^j d\vec{r} \cdot [\vec{A}(\vec{r}) - \vec{A}(\vec{a}+\vec{r})] = \int_i^j d\vec{r} \cdot [\vec{A}(\vec{a}+\vec{r}) - \vec{A}(\vec{r})]$$

soit $\chi_{\vec{a}}(\vec{r}) \equiv \int_0^{\vec{r}} d\vec{r}' \cdot [\vec{A}(\vec{a}+\vec{r}') - \vec{A}(\vec{r}')] \quad \text{i.e.} \quad \vec{\nabla} \chi_{\vec{a}}(\vec{r}) = \vec{A}(\vec{r}-\vec{a}) - \vec{A}(\vec{r})$

$$\begin{aligned} \chi_{\vec{a}}(\vec{r}_i) - \chi_{\vec{a}}(\vec{r}_j) &= \int_0^i d\vec{r} \cdot [\vec{A}(\vec{a}+\vec{r}) - \vec{A}(\vec{r})] - \int_0^j d\vec{r} \cdot [\vec{A}(\vec{a}+\vec{r}) - \vec{A}(\vec{r})] \\ &= \int_i^j d\vec{r} \cdot [\vec{A}(\vec{a}+\vec{r}) - \vec{A}(\vec{r})] \\ &= + \int_i^j d\vec{r} \cdot [\vec{A}(\vec{r}) - \vec{A}(\vec{a}+\vec{r})] \quad \text{ok.} \end{aligned}$$

cette phase de la transfo de jauge dépend de la translation \vec{a} , du potentiel vecteur (i.e. de la jauge) \vec{A} et de la position \vec{r} (car c'est une transfo de jauge!).

donc $\underbrace{e^{i2\pi\chi_{\vec{a}}(\vec{r})} \hat{T}_{\vec{a}} \hat{H} \hat{T}_{-\vec{a}} e^{-i2\pi\chi_{\vec{a}}(\vec{r})}}_{= \hat{H}}$

$$\underbrace{e^{i2\pi\chi_{\vec{a}}(\vec{r})} \hat{T}_{\vec{a}} |\psi\rangle}_{\hat{\mathcal{T}}_{\vec{a}} |\psi\rangle}$$

$$= E \hat{\mathcal{T}}_{\vec{a}} |\psi\rangle$$

i.e. $\hat{\mathcal{T}}_{\vec{a}} \hat{H} \hat{\mathcal{T}}_{-\vec{a}} = \hat{H}$

i.e. $[\hat{\mathcal{T}}_{\vec{a}}, \hat{H}] = 0$

→ générateur de translation magnétique.

translation magnétique = translation + transfo de jauge

$$\hat{G}_{\vec{a}} |\psi\rangle = e^{i2\pi \chi(\vec{r})} \frac{1}{T_{\vec{a}}} |\psi\rangle$$

$$\langle \vec{r} | \hat{G}_{\vec{a}} |\psi\rangle = e^{i2\pi \chi(\vec{r})} \underbrace{\langle \vec{r} | \frac{1}{T_{\vec{a}}} |\psi\rangle}_{\psi(\vec{r} + \vec{a})} = e^{i2\pi \int_0^{\vec{r}} d\vec{r}' \cdot [\vec{A}(\vec{r}' + \vec{a}) - \vec{A}(\vec{r}')] } \psi(\vec{r} + \vec{a})$$

$\chi_{\vec{a}}(\vec{r})$

~~$e^{i2\pi \int_0^{\vec{r}} d\vec{r}' \cdot \vec{A}(\vec{r}') - \vec{A}(\vec{r} + \vec{a})}$~~

ex: Réseau carré $N_x \times N_y$

flux de Landau $\vec{A} = Bx \vec{u}_y$

$\vec{a} = N_x \vec{u}_x$ $\vec{b} = N_y \vec{u}_y$

$$f = \frac{Ba^2}{\phi_0} = B$$

$$2\pi f = 2\pi B \equiv \theta$$

phases sur les contours ?

$$\left. \begin{aligned} \vec{A}(\vec{r} + \vec{a}) - \vec{A}(\vec{r}) &= B(x + N_x) \vec{u}_y - Bx \vec{u}_y = -B N_x \vec{u}_y \\ 2\pi \int_0^{\vec{r}} d\vec{r}' \cdot [\vec{A}(\vec{r}' + \vec{a}) - \vec{A}(\vec{r}')] &= -2\pi f N_x \int_0^{\vec{r}} d\vec{r}' \cdot \vec{u}_y = -2\pi f N_x x \end{aligned} \right\} \text{direction horizontale}$$

$$\left. \begin{aligned} \vec{A}(\vec{r} + \vec{b}) - \vec{A}(\vec{r}) &= \cancel{B(x + N_x) \vec{u}_y} - Bx \vec{u}_y = 0 \end{aligned} \right\} \text{direction verticale.}$$

Cela colle bien avec ce que j'avais traité le 10/03/2016. A savoir que sur les contours verticaux il n'y avait rien à changer et que sur les contours horizontaux il faut mettre une phase

il faut mettre une phase $\chi_{y=j-1} = -2\pi f N_x (j-1)$

~~$\vec{A} = B(x + N_x) \vec{u}_y$~~