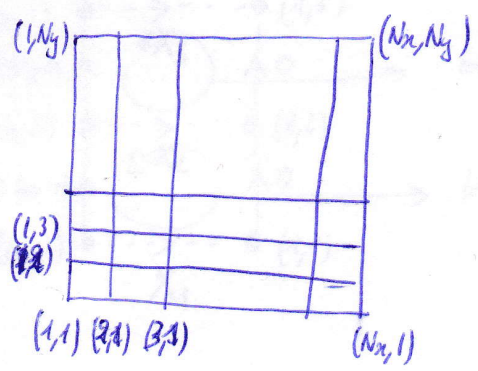


10/03/2016

Hofstadter butterfly on the square lattice with PBC and N values of the flux of there are N sites

Square lattice TB in $\perp \vec{B}$ field ~~and PBC~~

1) OBC



total # of sites = $N = N_x N_y$

$\vec{r} = (x, y) = (i-1, j-1)a \Leftrightarrow (i, j)$

Landau gauge $\vec{A} = Bx\hat{y}$

$t = t_{ij} \rightarrow t_{ij} = t e^{i \frac{e}{\hbar} \int_i^j d\vec{e} \cdot \vec{A}}$
 $= e^{i 2\pi \int_i^j d\vec{e} \cdot \vec{A}}$

$e \equiv 2\pi, \hbar \equiv 1$ such that $\phi_0 = 1$
 $\alpha \equiv 1$
 $t \equiv 1$

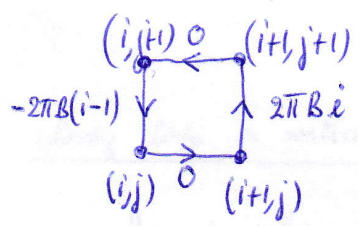
$f \equiv \frac{Ba^2}{\phi_0} = B$ flux per unit cell

$(i, j+1)$
 \uparrow
 $(i, j) = e^{i 2\pi B x_i} = e^{i 2\pi B (i-1)}$

phase = $2\pi B (i-1)$

$(i, j) \rightarrow (i+1, j) = e^{i 0} = 1$

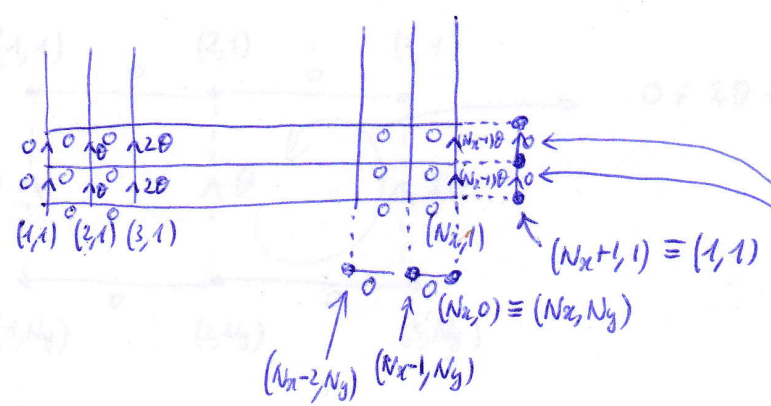
phase = 0



tot. phase = $0 + 2\pi B i + 0 - 2\pi B (i-1) = 2\pi B$

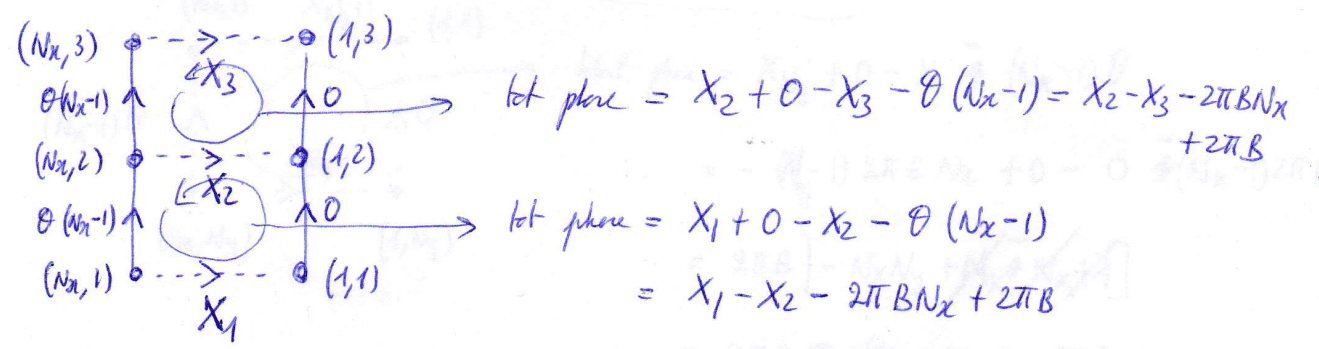
total flux = B

2) PBC



we have to decide what to put as a phase on the extra links ---
 All the existing links already have a phase including,

closing links in horizontal direct° (with phase X_j on the vertical direct°)



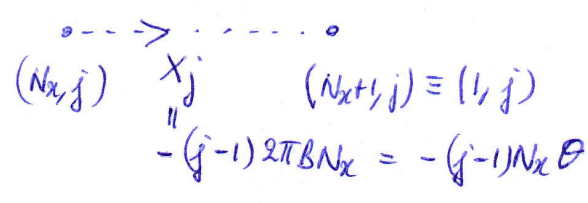
$\Rightarrow X_j - X_{j+1} - 2\pi B N_x + 2\pi B = \text{tot phase} = 2\pi B \pmod{2\pi}$

for $j = 1$ to N_y

e.g. $X_j = -(j-1)2\pi B N_x$

then $X_j - X_{j+1} - 2\pi B N_x + 2\pi B = \cancel{-(j-1)2\pi B N_x} + \cancel{-(j)2\pi B N_x} + 2\pi B$
 $= 2\pi B [-(j-1)N_x + jN_x - N_x + 1]$
 $= 2\pi B \text{ OK.}$

so all horizontal links have phase = 0 except the ones closing the torus that have



closing links in vertical direct°

there is nothing to do?

