

# Quantum Gravity at the Planck Scale:

## Getting a Handle on 'Spacetime Foam'

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## Mission Statement

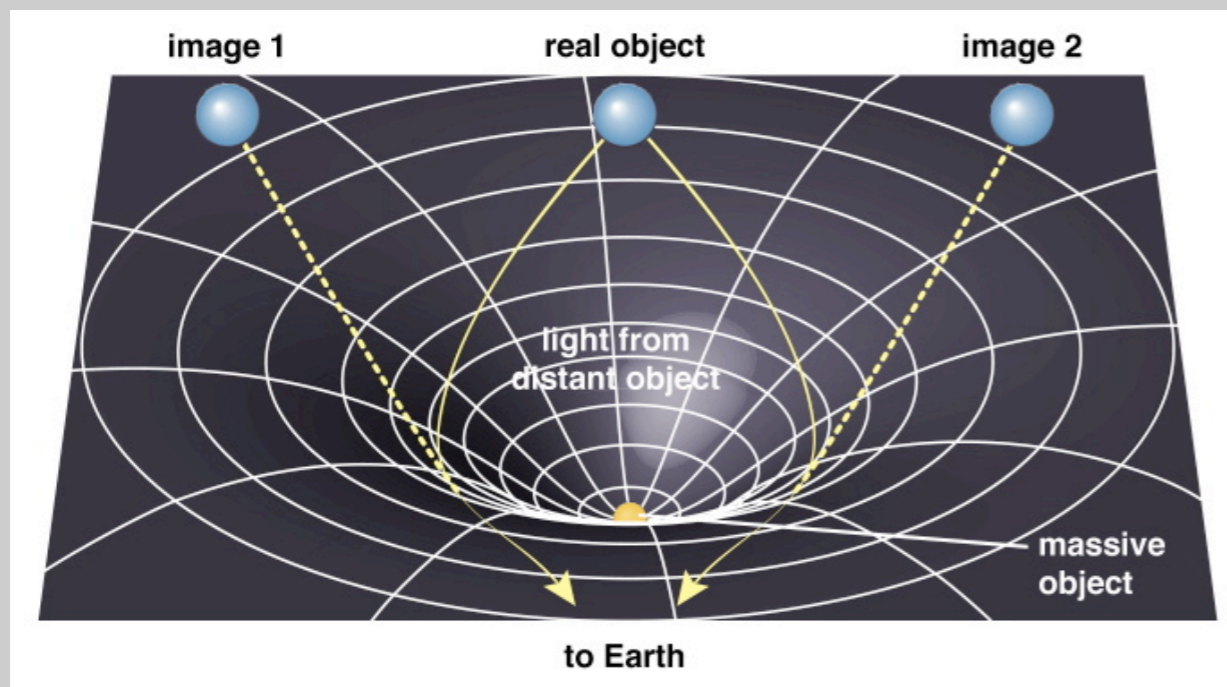
To find a theory of Quantum Gravity, that is, to find the quantum (field) theory underlying the classical theory of General Relativity.

In other words, to understand and describe quantitatively the structure and dynamics of spacetime on all scales.

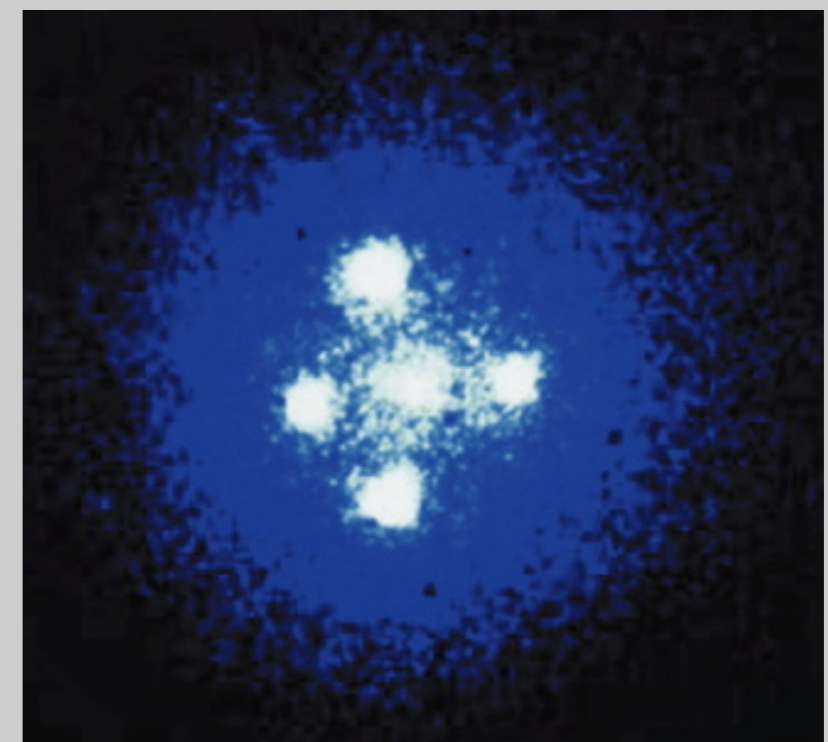
# Classical gravity = General Relativity



Einstein's 1915 theory unifies the notions of space, time and gravity into the single concept of a *curved spacetime*. His profound insight was that because of the universal character of gravity, all gravitational interactions (e.g. attraction of massive bodies, bending of light and particle trajectories, dynamics of the large-scale universe, ...) can be encoded in the local curvature structure of empty spacetime itself.



a gravitational lense  
(G2237 + 305)



Empty, curved spacetime can bend light rays which pass through it.

## A lesson from 20th century physics

Contrary to what our everyday intuition may suggest, empty spacetime is in itself an interesting entity with intricate dynamical, local curvature properties.

How spacetime curves and moves, depending on its matter/energy content, and subject to boundary conditions, is described by the Einstein equations of General Relativity (“classical gravity”), formulated in terms of a Lorentzian metric  $g_{\mu\nu}(x)$  of indefinite signature  $(-+++)$ ,

$$R_{\mu\nu}[g, \partial g, \partial^2 g](x) - \frac{1}{2}\Lambda g_{\mu\nu}(x) = 8\pi T_{\mu\nu}(x)$$

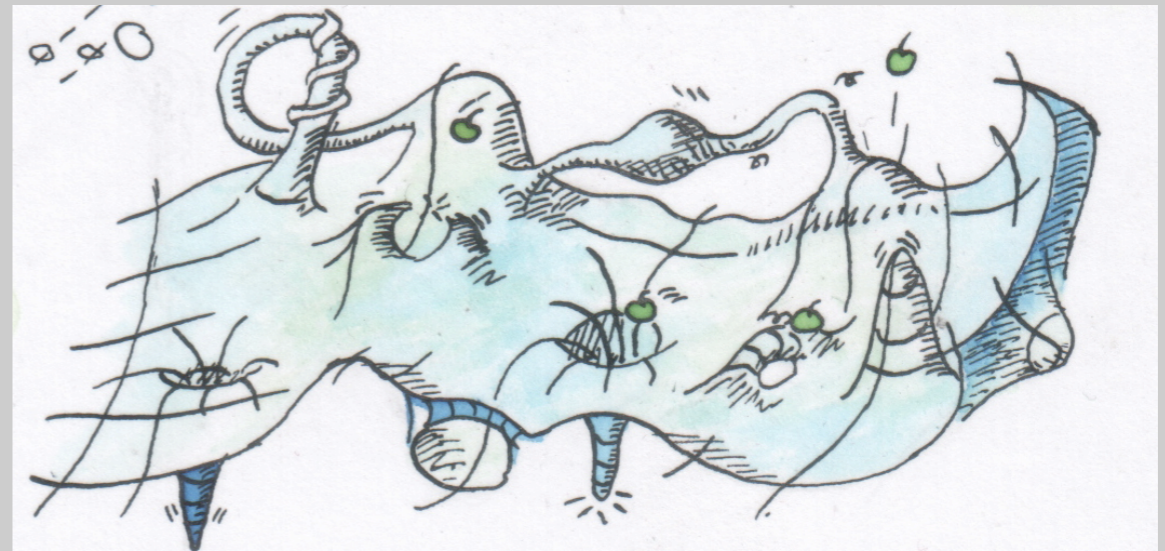
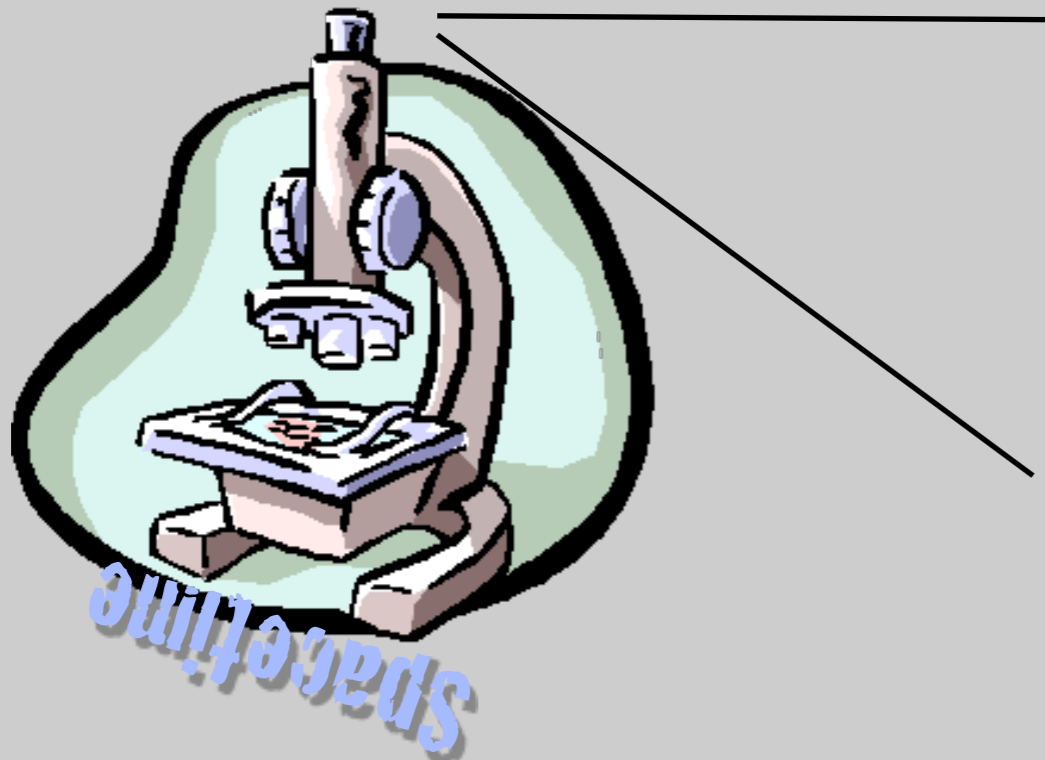
The physical curvature properties of spacetime are invariant under a change of coordinates  $x^\mu = y^\mu(x^\nu)$ , a “diffeomorphism”. Therefore, the “habitat” of General Relativity is the space of geometries on a differentiable 4-manifold  $M$ ,

$$\mathcal{G}(M) = \text{Lor}(M) / \text{Diff}(M)$$

## But this is not the end of it ...

Einstein's classical theory of General Relativity cannot answer all questions we may ask about empty spacetime - it is incomplete!

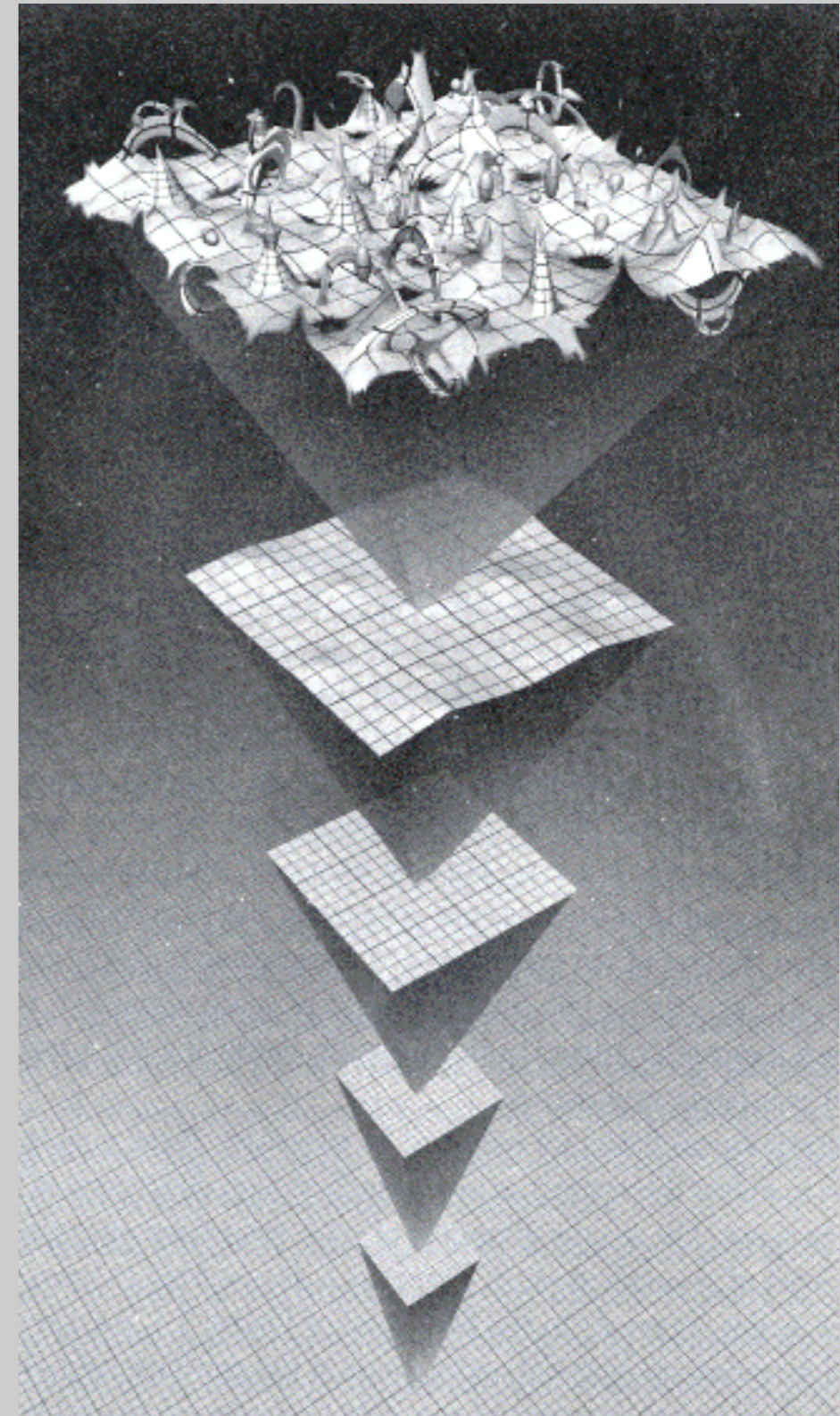
- It is well tested from cosmological scales down to the millimeter range.
- On atomic/nuclear scales gravity appears to be completely unimportant.
- On even smaller scales, eventually reaching the Planck scale, spacetime must be described by *quantum*, and not by *classical* equations of motion.
- Combining general arguments from quantum theory and general relativity, we expect that the microstructure of spacetime near the Planck scale is completely dominated by quantum fluctuations.



Spacetime Foam?

# Some unresolved fundamental questions

- What are the (quantum) origins of space, time and our universe?
- What is the microstructure of spacetime?
- What are the relevant degrees of freedom at the Planck scale?
- Can their dynamics *explain* the observed large-scale structure of our universe, that of an approximate ~~Minkowski~~ de Sitter space?
- Which aspects of spacetime are dynamical at the Planck scale: geometry? topology? dimensionality?
- Are “space”, “time”, and “causality” fundamental or emergent?



# An ultrabrief history of quantum gravity

classical gravity = solving Einstein's equations

→ a classical spacetime  $(M, g_{\mu\nu}(x))$

quantum gravity = solving “quantum Einstein equations”

→ a “quantum spacetime” ???

We can distinguish between grand unified approaches (like superstring theory) and approaches which treat gravity and matter on a distinct footing, akin to what happens in the classical theory.

In the non-unified approaches, one traditionally distinguishes between “canonical” and “covariant” methods. In the former, one performs an explicit split of time and space and quantizes certain algebras of observables in terms of selfadjoint operators (e.g. loop quantum gravity). In the latter, one quantizes using path integrals, and keeps the spacetimes intact.

There is a perturbative version of the gravitational path integral, which because of its perturbative nonrenormalizability does not lead to a quantum theory valid on all scales. My work uses a *nonperturbative* path integral.

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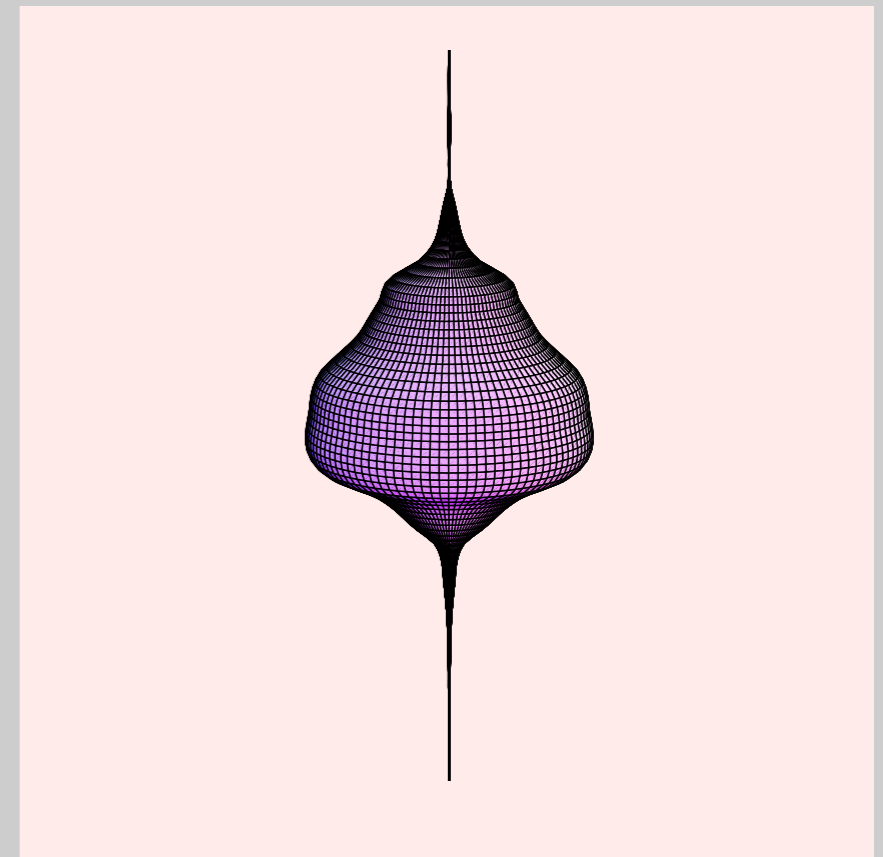
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# Quantum Gravity from Causal Dynamical Triangulation (QG from CDT)★

CDT is a no-frills nonperturbative implementation of the gravitational path integral, much in the spirit of lattice quantum field theory, but based on *dynamical* lattices, reflecting the dynamical nature of spacetime geometry.

A key result that puts QG from CDT on the map as a possible quantum theory of gravity is the fact that it can generate dynamically a background geometry with semiclassical properties from pure quantum excitations, in an a priori background-independent formulation. (C)DT has also given us crucial *new* insights into nonperturbative dynamics and pitfalls.

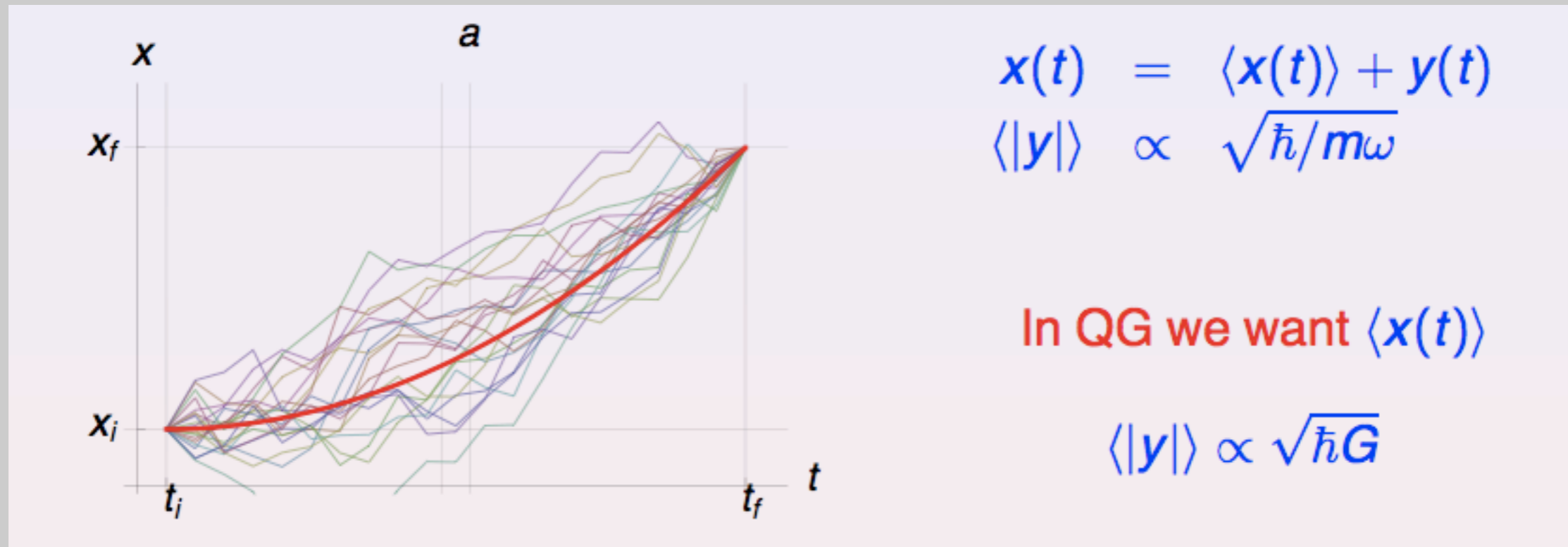


(PRL 93 (2004) 131301, PRD 72 (2005) 064014, PLB 607 (2005) 205)

★ my presentation today is mainly based on joint work with  
J. Ambjørn, J. Jurkiewicz, T. Budd, A. Görlich and S. Jordan

# Basic tool: the good old path integral, or, from classical to quantum trajectories

Textbook example: the nonrelativistic particle (h.o.) in one dimension



Quantum superposition principle: the transition amplitude from  $x_i(t_i)$  to  $x_f(t_f)$  is given as a weighted sum over amplitudes  $\exp iS[x(t)]$  of all possible trajectories, where  $S[x(t)]$  is the classical action of the path.

(here, time is discretized in steps of length  $a$ , and the trajectories are piecewise linear)

# The same superposition principle, applied to gravity

"Sum over histories"  
a.k.a. gravitational path integral

$$Z(G_N, \Lambda) = \int \mathcal{D}g e^{iS_{G_N, \Lambda}^{EH}[g]}$$

cosmol. const.  $\downarrow$   
Newton const.  $\rightarrow$   $G_N$  spacetime geom.s  $g \in \mathcal{G}$

Each "path" is now a four-dimensional, curved spacetime geometry  $g$ , which can be thought of as a three-dimensional, spatial geometry developing in time. The weight associated with each  $g$  is given by the corresponding Einstein-Hilbert action  $S^{EH}[g]$ ,

$$S^{EH} = \frac{1}{G_N} \int d^4x \sqrt{-\det g} (R[g, \partial g, \partial^2 g] - 2\Lambda)$$

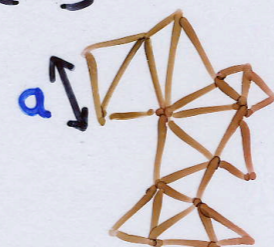
How can we make  $Z(G_N, \Lambda)$  into a meaningful, well-defined quantity?

# Regularizing the path integral via CDT

Sum over histories  $Z(G_N, \Lambda)$ :

$$\int Dg \, e^{iS^{EH}[g]} \stackrel{\text{CDT}}{=} \lim_{\substack{a \rightarrow 0 \\ N \rightarrow \infty}} \sum_{\substack{\text{inequiv.} \\ \text{triangul.s} \\ T \in \mathcal{G}_{a,N}}} \frac{1}{C(T)} e^{iS^{\text{Regge}}[T]}$$

$\int Dg$  spacetime geom.s  $g \in \mathcal{G}$   
 $e^{iS^{EH}[g]}$  curved spacetime geometry  $g$   
 $\stackrel{\text{CDT regular.}}{\longrightarrow}$  gluing  $T$  of  $N$  simplices (piecewise flat manifold)  
 $\frac{1}{C(T)}$   $|Aut(T)|$   
 $a$

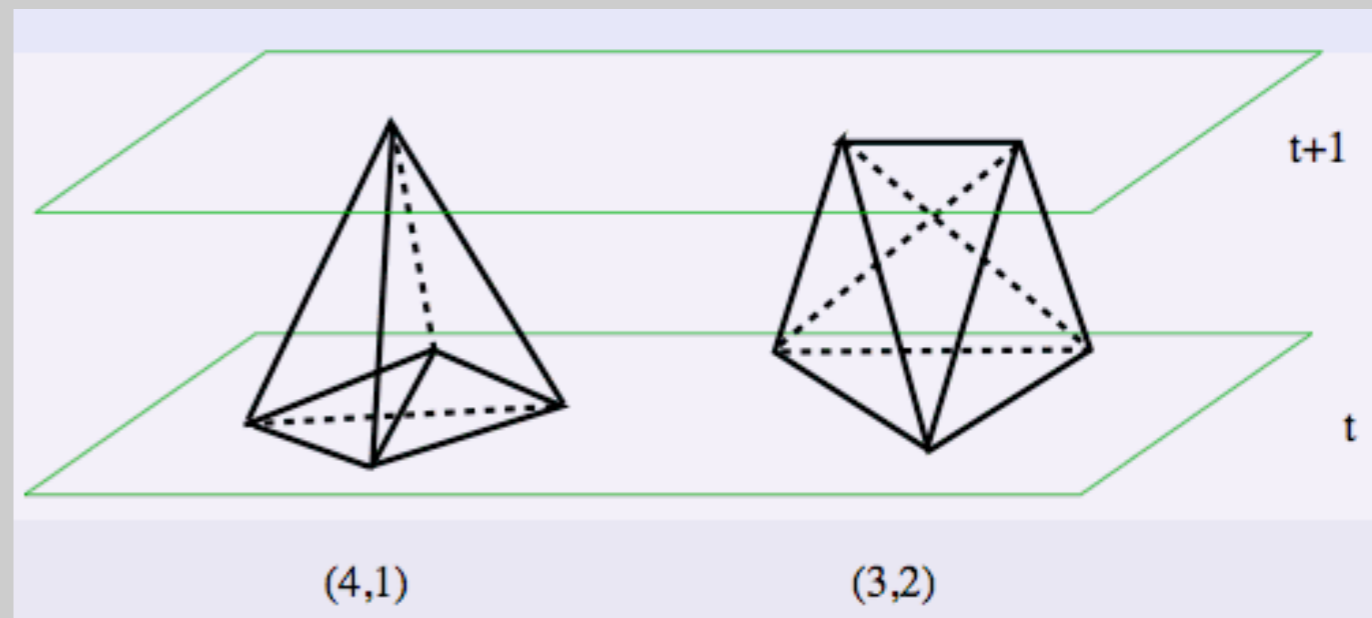


‘democratic’, regularized sum over piecewise flat spacetimes, doesn’t need coordinates (Regge); continuum limit required to obtain universal results independent of the regularization

Elementary four-simplex, building block for a causal dynamical triangulation:

$a \sim$  edge length; diffeomorphism-invariant UV regulator

Micro-causality is essential! This does not work in Euclidean signature - get only branched polymers (~mid-90s).



CDT’s proper-time slicing

# Wick rotation and analogy with statistical mechanics

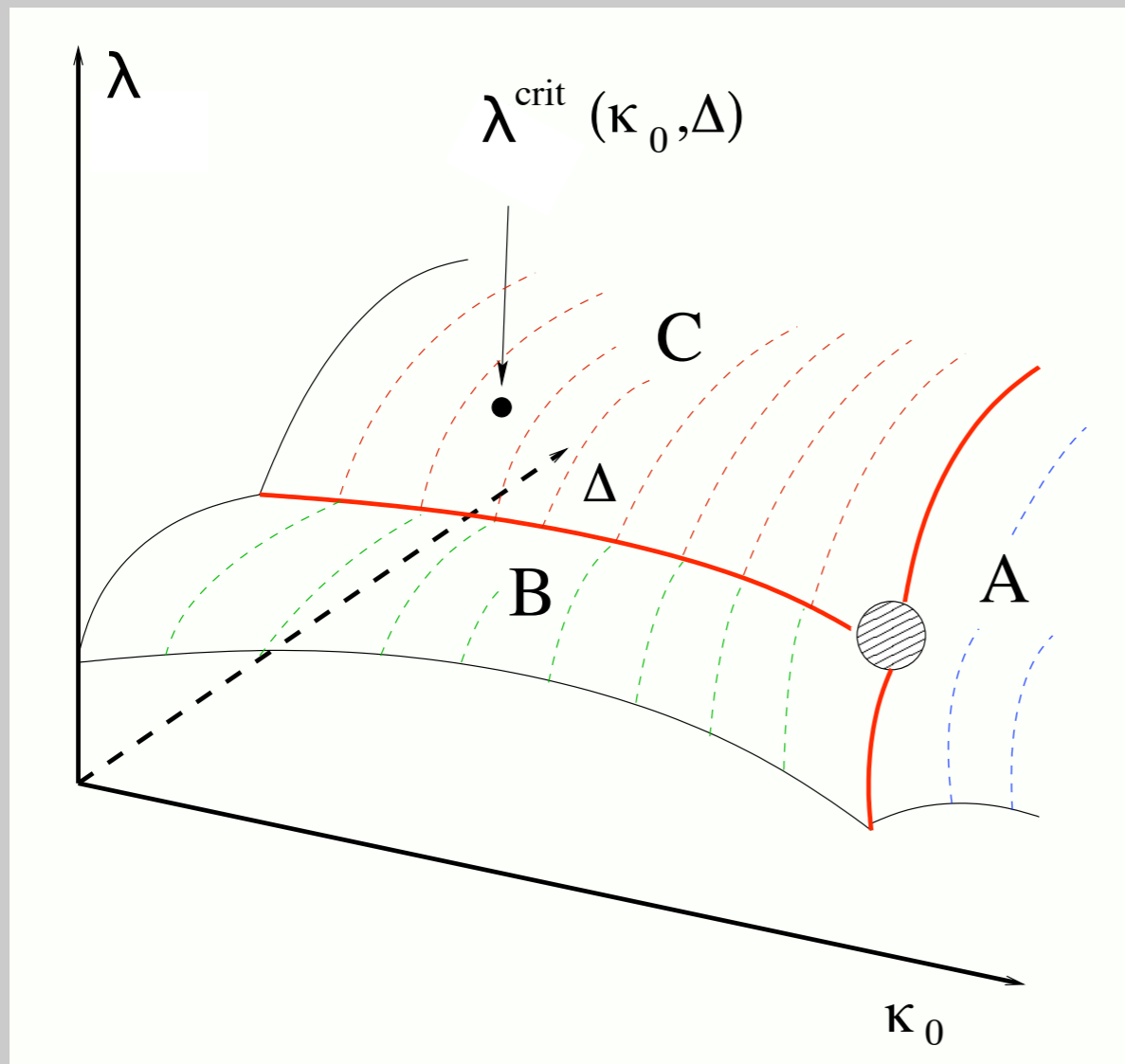
- each regularized Lorentzian geometry  $T$  allows for a rotation to a unique regularized Euclidean geometry  $T_{\text{eu}}$ , such that the Feynman amplitude of a path is turned into a Boltzmann weight, as in statistical mechanics

$$e^{iS^{\text{Regge}}(T)} \rightarrow e^{-S_{\text{eu}}^{\text{Regge}}(T_{\text{eu}})}$$

- this turns the quantum amplitude  $Z$  into a partition function  $Z_{\text{eu}}$  and allows us to use powerful numerical methods from statistical mechanics, like Monte Carlo simulations
- a ‘classical trajectory’ is an average over quantum trajectories in the statistical ensemble of trajectories (the Euclideanized ‘sum over histories’)
- taking the continuum limit of this regularized theory means studying the critical behaviour of the underlying statistical theory
- performing an ‘inverse Wick rotation’ on quantities computed in the continuum limit is in general nontrivial

$$S^{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-\det g} (R[g, \partial g, \partial^2 g] - 2\Lambda)$$

# The phase diagram of Causal Dynamical Triangulations



$$S_{\text{eu}}^{\text{Regge}} = -\kappa_0 N_2 + N_4(c\kappa_0 + \lambda) + \Delta(2N_4^{(4,1)} + N_4^{(3,2)})$$

$\lambda$  ~ cosmological constant

$\kappa_0$  ~  $1/G_N$  inverse Newton's constant

$\Delta$  ~ relative time/space scaling

$c$  ~ numerical constant,  $>0$

$N_i$  ~ # of triangular building blocks of dimension  $i$

The partition function is defined for  $\lambda > \lambda^{\text{crit}}(\kappa_0, \Delta)$ ;  
 approaching the critical surface = taking infinite-volume limit.

red lines ~ phase transitions

(J. Ambjørn, J. Jurkiewicz, RL, PRD 72 (2005) 064014;

J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, PLB 690 (2010) 413;

definite phase transition analysis: work with S. Jordan, to appear)

# Why we are so excited about this approach, or, The dynamical emergence of spacetime as we know it

CDT is the so far only candidate theory of nonperturbative quantum gravity where a classical extended geometry is generated from nothing but Planck-scale quantum excitations.

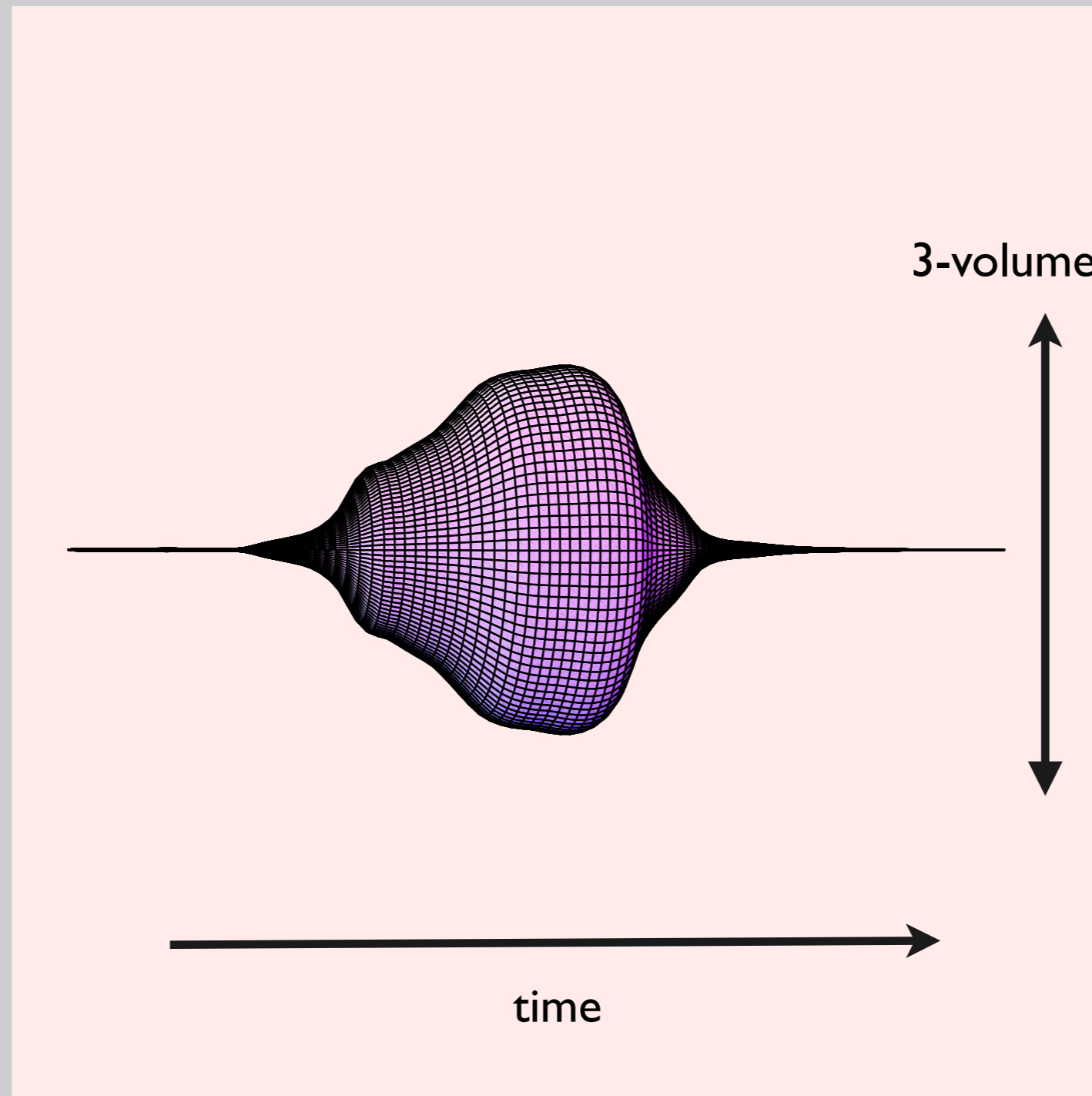
This happens by a nonperturbative, entropic<sup>★</sup> mechanism:

Magically, the many microscopic building blocks in the quantum superposition arrange themselves into an extended quantum spacetime whose macroscopic shape is that of a well known cosmology.

When, from all the gravitational degrees of freedom present, we monitor only the spatial three-volume  $\langle V_3(t) \rangle$  of the universe as a function of proper time  $t$ , we find a distinct “volume profile”.

★entropy = number of microscopic geometric realizations of a given value of the action

# Dynamically generated four-dimensional quantum universe, obtained from a path integral over causal spacetimes



This is a Monte Carlo “snapshot” - still need to average to obtain the *expectation value* of the volume profile.



# Our “self-organized quantum spacetime” has the shape of a de Sitter universe!



(A solution to the classical Einstein equations in the presence of “dark energy” - a.k.a. a cosmological constant  $\Lambda$ .)

Classical de Sitter space has

$$V_3(\tau) = 2\pi^2 \left( c \cosh \frac{\tau}{c} \right)^3$$

$c$  constant

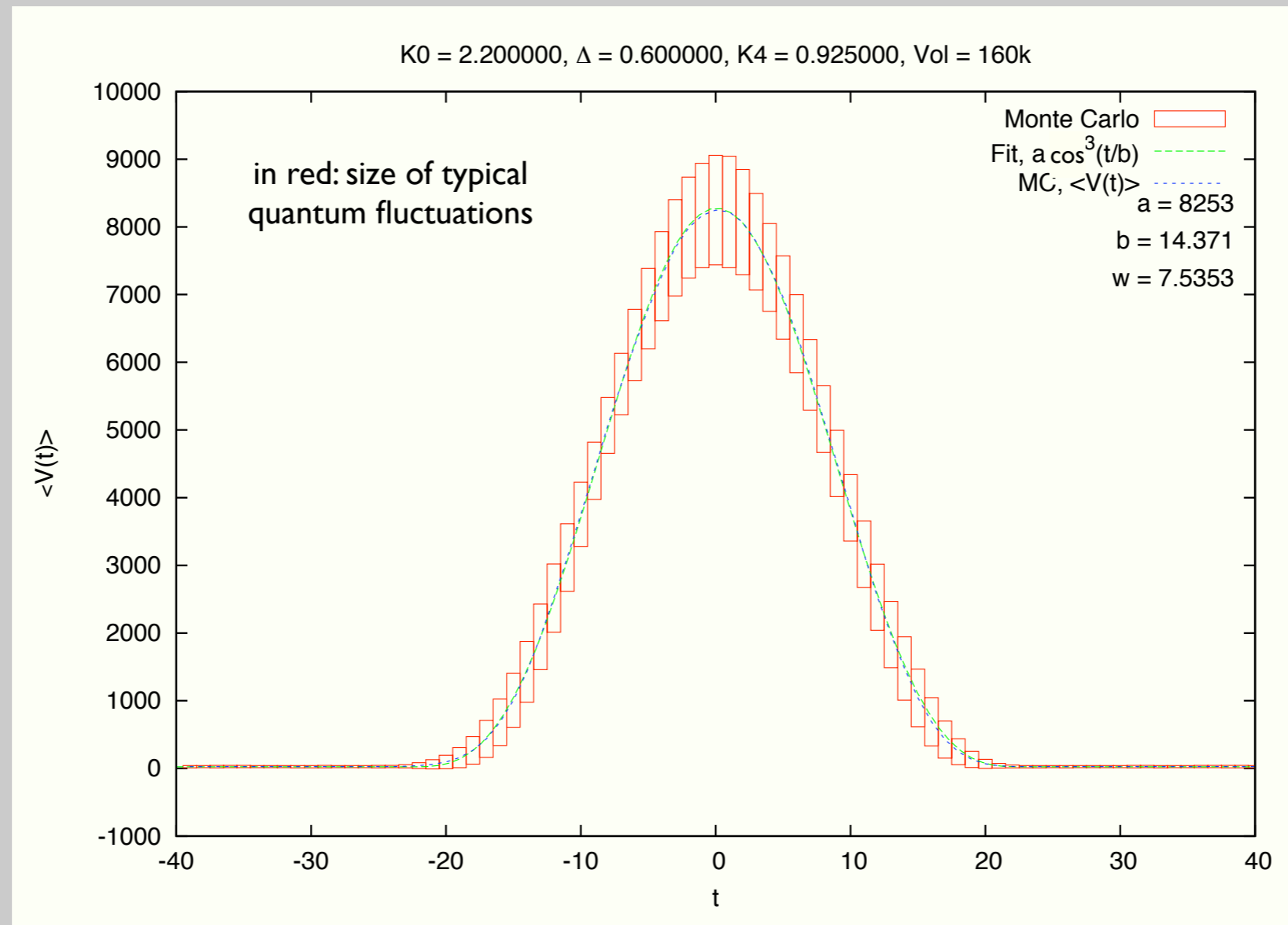
giving rise to an exponentially expanding universe,  $V_3 \sim e^{c\tau}$ , for  $\tau > 0$ .

- We measure this for Euclidean time  $t = i\tau$

➔ a very nontrivial test of the classical limit;  
strong flavour of condensed matter phenomena

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PRL 100 (2008) 091304, PRD 78 (2008) 063544, NPB 849 (2011) 144 (w/ J. Gizbert-Studnicki, T. Trzesniewski)

# What is the concrete evidence for de Sitter space?



The volume profile  $\langle V_3(t) \rangle$ , as function of Euclidean proper time  $t=i\tau$ , perfectly matches that of a Euclidean *de Sitter space*, with scale factor  $a(t)^2$  given by

$$ds^2 = dt^2 + a(t)^2 d\Omega_{(3)}^2 = dt^2 + c^2 \cos^2 \left( \frac{t}{c} \right) d\Omega_{(3)}^2 \leftarrow \text{volume el. } S^3$$

N.B.: we are *not* doing quantum cosmology

# Getting a handle on Planckian physics

Also the analysis of the *short-scale* structure of the universe proceeds via ‘computer experiment’, where we measure the expectation values of suitable quantum *observables*  $\hat{\mathcal{O}}$  according to

$$\langle \hat{\mathcal{O}} \rangle = \frac{\int \mathcal{D}g \mathcal{O} e^{-S[g]}}{Z}, \quad Z = \int \mathcal{D}g e^{-S[g]}$$

N.B.: because of diffeomorphism invariance, observables cannot depend on coordinates, which means that any local  $\hat{\phi}(x)$  does *not* qualify. Observables which are well defined tend to be nonlocal, e.g. certain spacetime integrals.

A distinguished class of observables which probe the average local geometry of quantum spacetime are certain notions of “dimension”, first used in the context of lower-dimensional (DT) toy models of quantum gravity.

Remarkably, in such models “dimension” becomes dynamical and can exhibit anomalous scaling, and is not *a priori* determined by the dimension at the cut-off scale. (H. Kawai, N. Kawamoto, T. Mogami, Y. Watabiki, PLB 306 (1993) 19; J. Ambjørn, Y. Watabiki, NPB 445 (1995) 129).

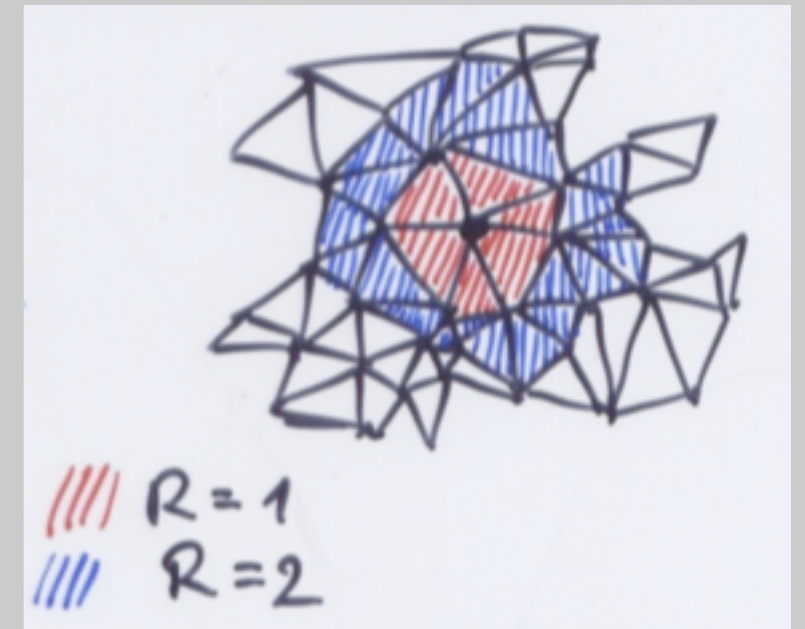
→ B. Duplantier’s talk

# Example I: the Hausdorff dimension

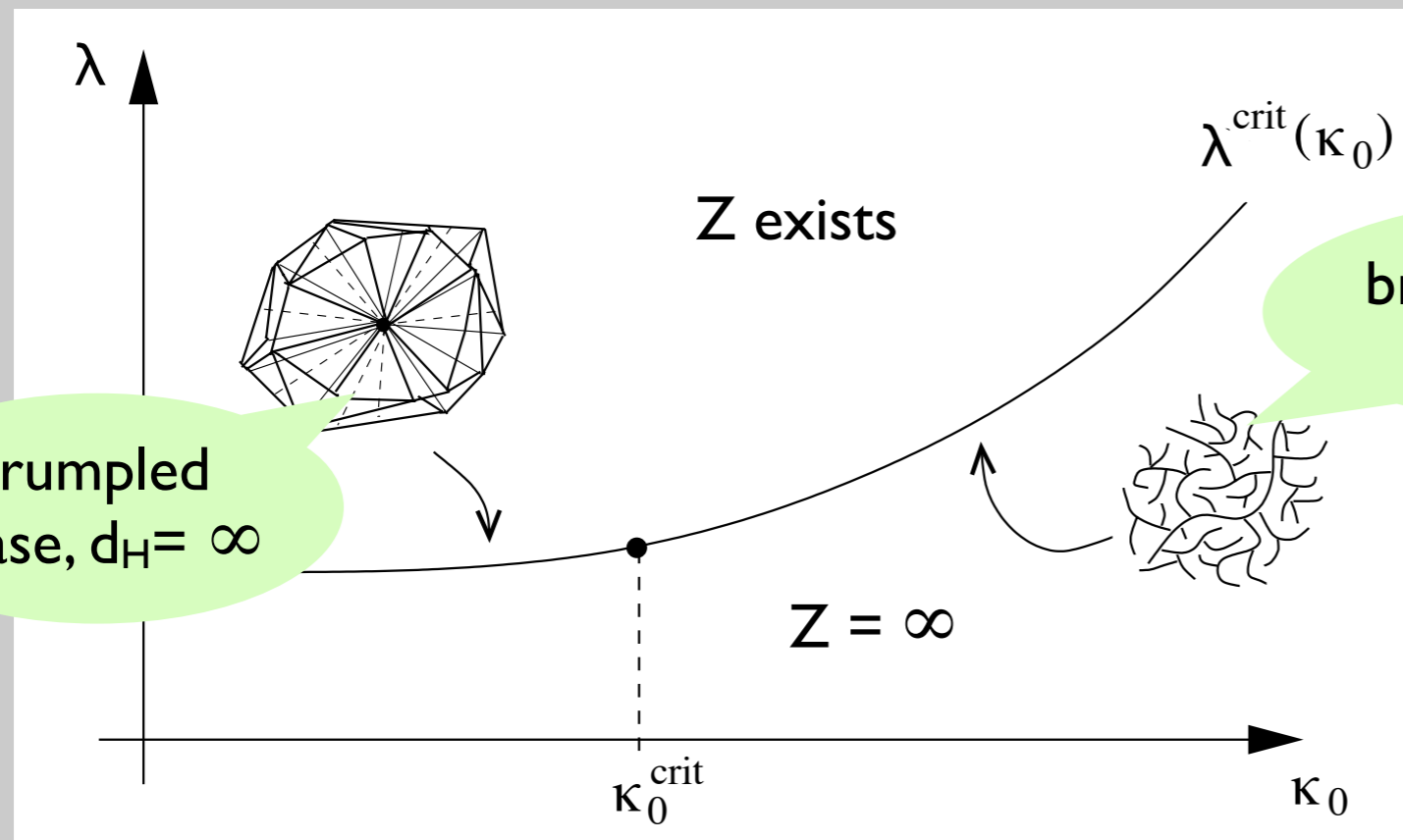
One way of characterizing quantum geometry in DT gravity models is through the Hausdorff dimension  $d_H$ , obtained by measuring the observable

$$\langle V(R) \rangle \sim R^{d_H}$$

where  $V(R)$  denotes the volume of a geodesic ball of radius  $R$  around some vertex, averaged over vertices. The (discrete) geodesic distance  $R$  is given by the number of edges in the shortest path connecting a pair of vertices, and  $V$  is the number of simplices.



measuring  $d_H$  in a 2D triangulation



branched-polymer phase,  $d_H=2$

crumpled phase,  $d_H = \infty$

phase diagram of Euclidean quantum gravity via dynamical triangulations in 4D (alas, it has no good classical limit).

## Example 2: the spectral dimension

(or, more nonperturbative surprises!)



Another way of probing the geometry of an unknown space is by way of a diffusion process. In particular, this will be sensitive to the dimension of the “medium” in which the “spreading” takes place,

$$\text{Volume}(\text{“ink cloud”}) \sim \sigma^{D_S/2},$$

where  $\sigma$  is the external diffusion time, and  $D_S$  the so-called *spectral dimension*.

Recall diffusion on  $d$ -dimensional flat space  $R^d$ :

$$P(x, x_0; \sigma) = \frac{e^{-(x-x_0)^2/4\sigma}}{(4\pi\sigma)^{d/2}} \quad \text{solves} \quad \partial_\sigma P = \vec{\nabla}^2 P$$

The average return probability is given by

$$\mathcal{R}_V(\sigma) := \frac{1}{V(M)} \int_M d^d x P(x, x; \sigma) \equiv \frac{1}{(4\pi\sigma)^{d/2}} \Rightarrow d = -2 \frac{d \log \mathcal{R}_V(\sigma)}{d \log \sigma}$$

What is important for us is that diffusion processes can be defined on much more general structures, e.g. curved Riemannian manifolds  $(M, g_{\mu\nu})$ , piecewise flat spaces (and ensembles thereof), graphs, fractals, ...

## Implementing diffusion on a $d$ -dim. simplicial DT manifold

Discrete evolution on a triangulation  $T$  is defined by

$$P_T(j, i_0; \sigma + 1) = \frac{1}{d + 1} \sum_{k \rightarrow j} P_T(k, i_0; \sigma), \quad P_T(i, i_0; \sigma = 0) = \delta_{i, i_0}$$

where the simplex with label  $i_0$  is the starting point of the diffusion,  $k$  labels the  $d+1$  neighbouring simplices of simplex  $j$ , and  $\sigma$  counts discrete diffusion steps.

Again, we can use the average return probability (and its ensemble average),

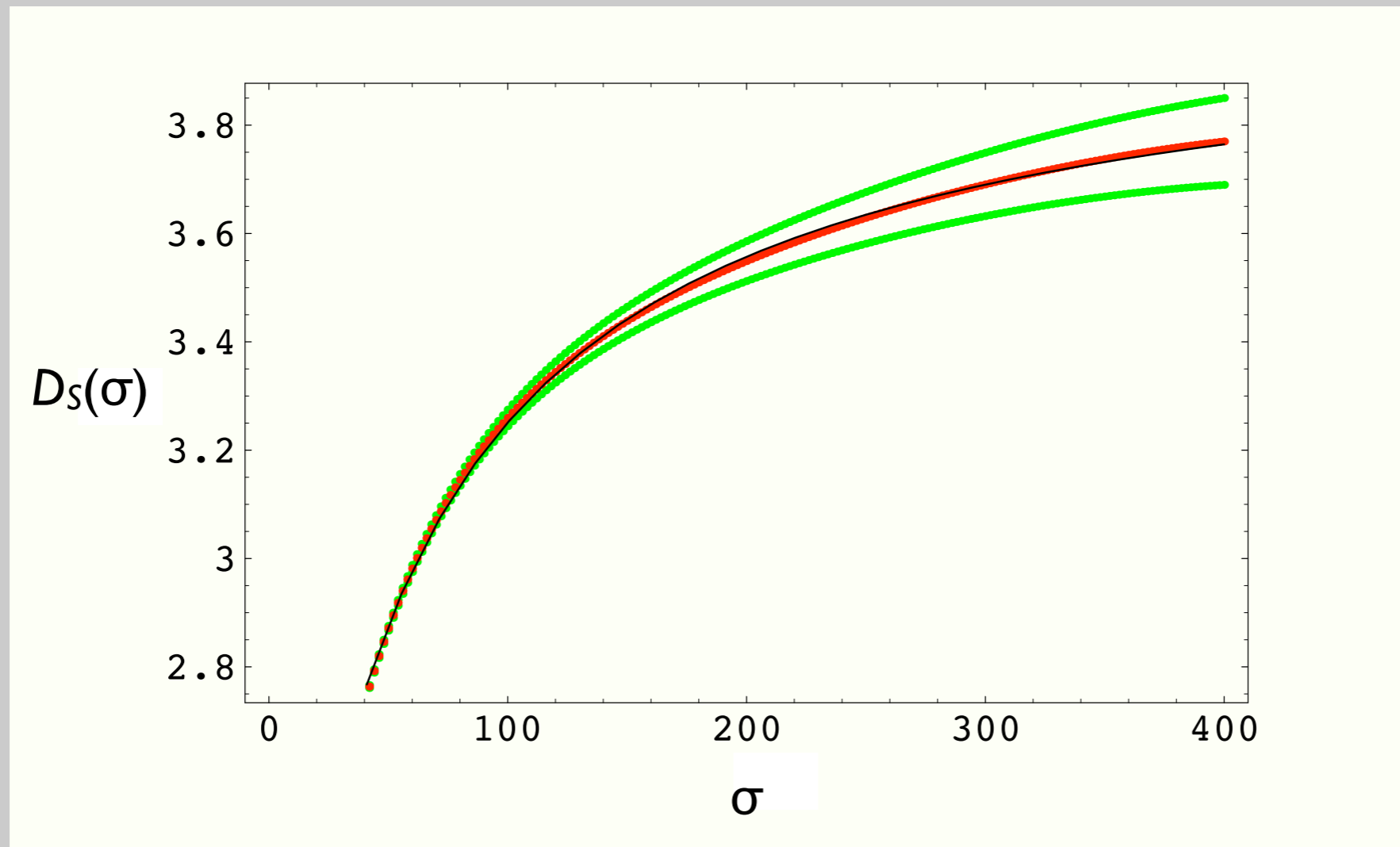
$$\mathcal{R}_T(\sigma) := \frac{1}{V(T)} \sum_{i_0 \in T} P_T(i_0, i_0; \sigma), \quad \langle \mathcal{R}(\sigma) \rangle_V = \frac{1}{Z_V} \sum_{T_V} \frac{1}{C_{T_V}} e^{-S^{\text{Regge}}(T_V)} \mathcal{R}_{T_V}(\sigma)$$

and *define* the corresponding spectral dimension as

$$D_S(\sigma) := -2 \frac{d \log \langle \mathcal{R}(\sigma) \rangle_V}{d \log \sigma}, \quad \sigma \lesssim V^{2/D_S}$$

$D_S(\sigma)$  probes quantum geometry at distances  $\sim \sigma^{1/2}$ . There is no a priori reason why this should coincide with the dimension of the triangular building blocks.

Quite remarkably, in CDT quantum gravity in 4D we find that  $D_S(\sigma)$  depends on the length scale:  $D_S$  changes smoothly from 4 on large scales to  $\sim 2$  on short scales. (J. Ambjorn, J. Jurkiewicz, RL, PRL 95 (2004) 171301)

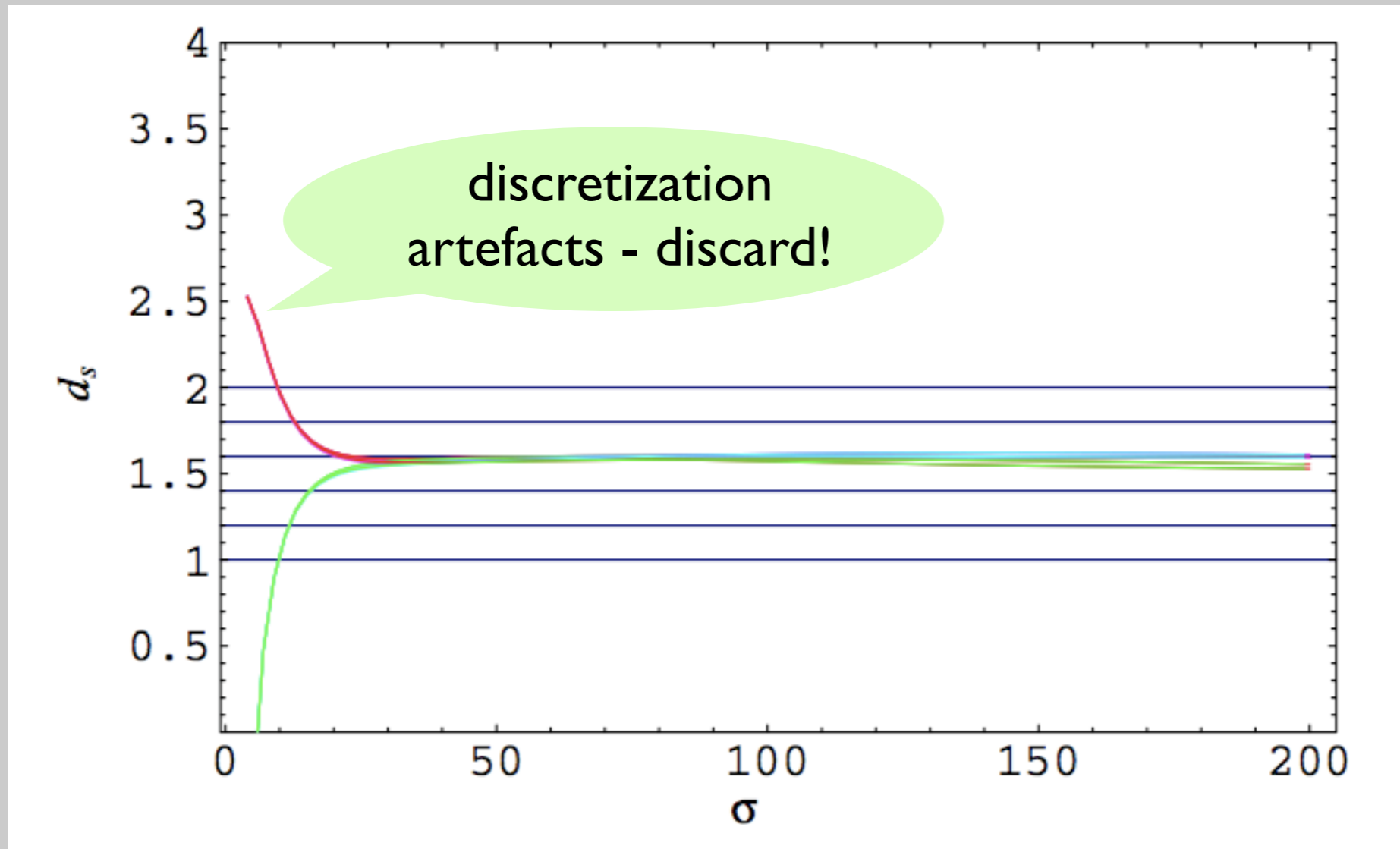


More precisely, we extrapolate from the shown infinite-volume limit that

$$D_S(\sigma) \rightarrow 4.02 \pm 0.1 \text{ as } \sigma \rightarrow \infty, \quad D_S(\sigma) \rightarrow 1.82 \pm 0.25 \text{ as } \sigma \rightarrow 0$$

Up to this point, measurements of various dimensions in dynamically triangulated models of quantum gravity had always thrown up fractal behaviour, in the sense of anomalous, but scale-independent dimensions.

This is even true when measuring the spectral dimension of spatial hypersurfaces  $t=const$  in 4D CDT quantum gravity:



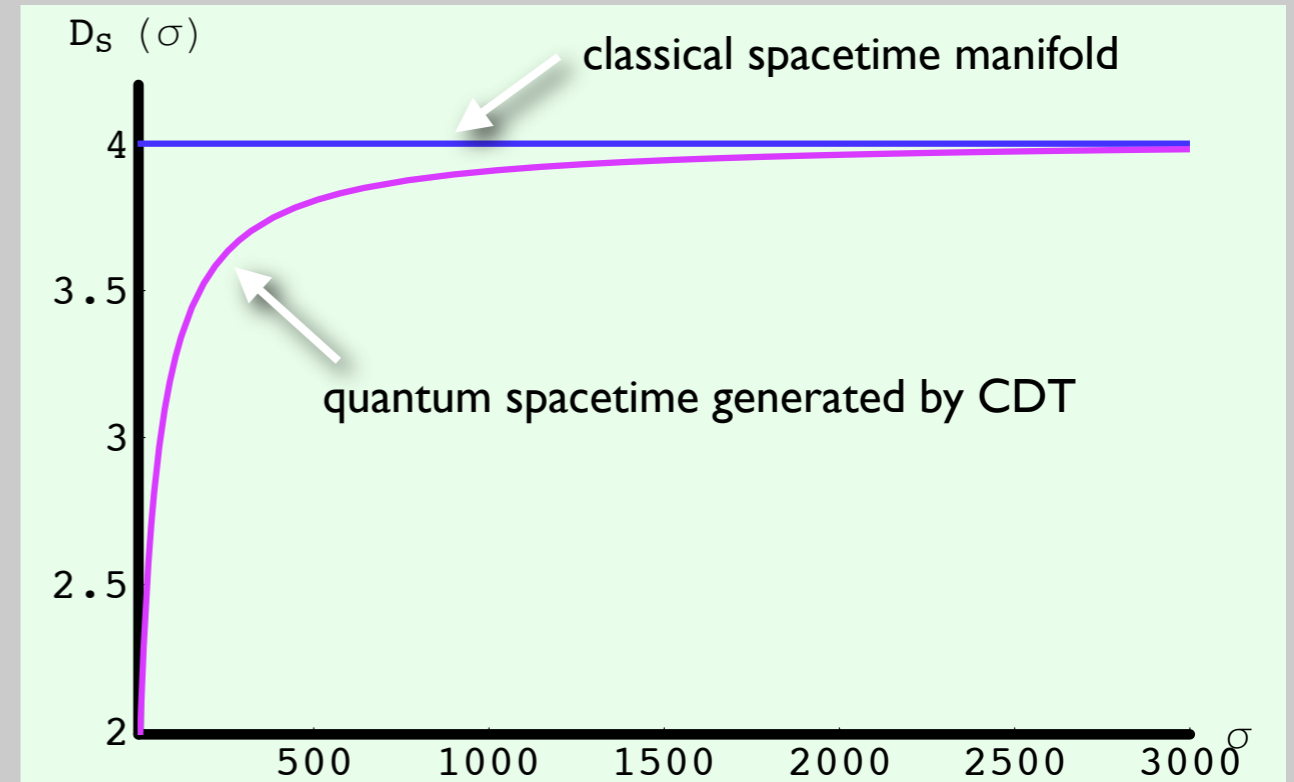
Logarithmic derivative of the return probability (at  $\kappa_0=2.2$ ,  $\Delta=0.4$ ,  $V_4=9|k$ ,  $t=40$ ,  $V_3=500\dots 2000$ ) (J. Ambjorn, J. Jurkiewicz, RL, PRD 72 (2005) 064014)



# Dynamical dimensional reduction in 4D quantum gravity

We conclude that on short scales, our “ground state of geometry” in 4D is definitely *not* a classical manifold.

This could also mean that nonperturbative quantum gravity has its own built-in ultra-violet regulator.



Intriguingly, a similar short-scale “*dynamical dimensional reduction*” has been found in a couple of disparate (but also quantum field-theoretic) approaches:

- nonperturbative renormalization group flow analysis (M. Reuter, O. Lauscher, JHEP 0510:050, 2005) → M. Reuter’s talk
- nonrelativistic “Lifshitz quantum gravity” (P. Hořava, PRL 102 (2009) 161301)

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-\det g} (R[g, \partial g, \partial^2 g] - 2\Lambda)$$

3d: relating the curve  $D_s(\sigma)$  of CDT (D. Benedetti, J. Henson, PRD 80 (2009) 124036) to dispersion relations of suitable differential operators on 3d flat space (T. Sotiriou, M. Visser, S. Weinfurtner, arXiv:1105.5646)

# Short-scale fractal structure of the quantum universe

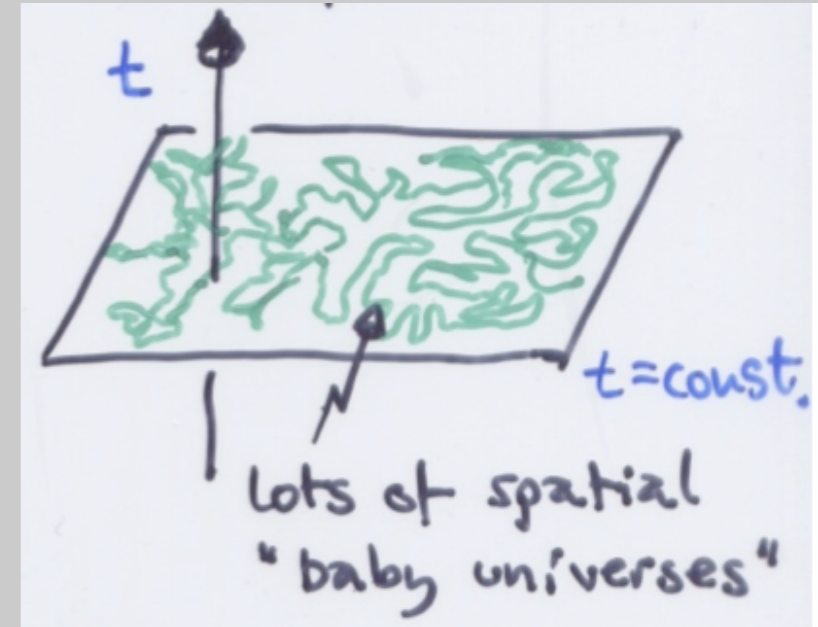
One idea: look at the geometric properties of hypersurfaces:

- compact slices of constant time,  $t=const.$

(J. Ambjorn, J. Jurkiewicz, RL, PRD 72 (2005) 064014)

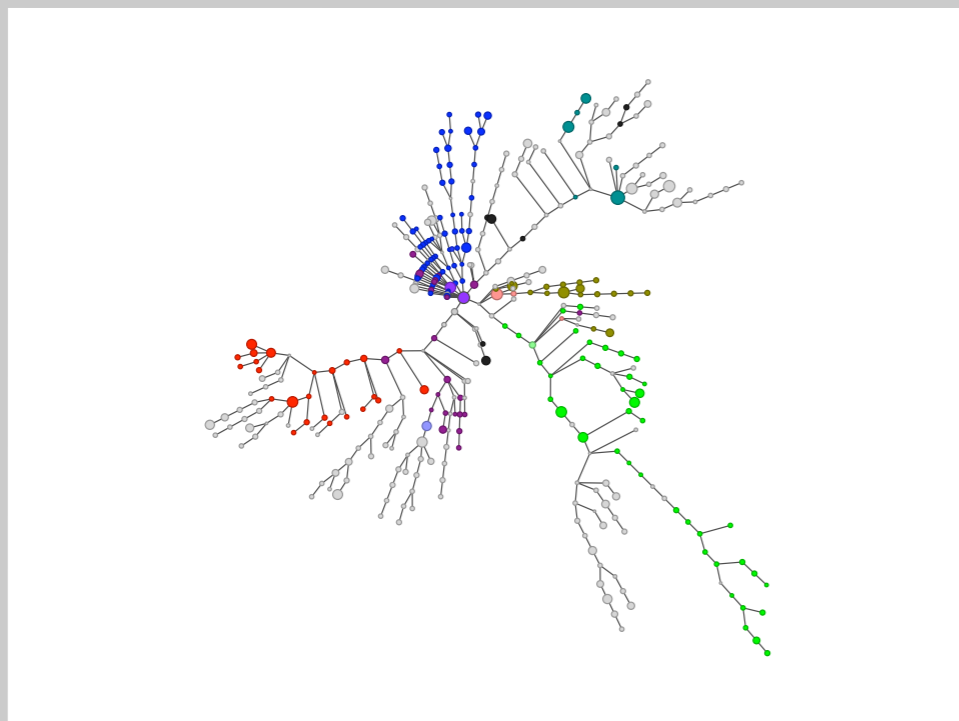
$$d_H \approx 3, \quad d_S \approx 3/2, \quad \gamma \approx 1/3,$$

$$\text{where } \mathcal{N}(N_3) \sim e^{\mu_0 N_3} N_3^{-3+\gamma}$$



- compact shells of constant radius,  $r=const.$  around some point  $x_0$

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PLB 690 (2010) 420)



The spatial slices of topology  $S^3$  have a tree structure in terms of smaller  $S^3$ -components separated by “minimal necks” (four triangles forming a minimal  $S^2$ , without the interior of the tetrahedron being part of the slice).

# The spectral dimension in quantum gravity

- $D_S(\sigma)$  is the “dimension felt by a scalar test particle” - not really a true observable near the Planck scale in the sense of phenomenological implications
- still useful and “covariantly defined” (meaningful in the sum over geometries and after averaging over the starting point of the diffusion process) and can be computed; characteristically nonlocal
- can play an important role in discriminating between different candidate theories of quantum gravity, akin to the computation of “black hole entropy”  $S=A/4$ , but arguably one that probes the nonperturbative structure, not just semiclassical properties
- various computations of  $D_S(\sigma)$  on short scales for nonclassical geometries: noncommutative geometry/ $\kappa$ -Minkowski space (D. Benedetti, PRL 102 (2009) 111303), three-dimensional CDT (D. Benedetti, J. Henson, PRD 80 (2009) 124036), from area operator in loop quantum gravity (L. Modesto, CQG 26 (2009) 242002), possible relation with strong-coupling limit of WdW equation (S. Carlip, arXiv: 1009.1136), modelling from dispersion relations on flat spaces (T. Sotiriou, M. Visser, S. Weinfurtner, arXiv: 1105.6098), modelling by multifractal spacetimes (G. Calcagni, arXiv: 1106.0295)

# The quest for observables

- We are urgently searching quantum observables to characterize the Planckian properties of spacetime, going beyond the various notions of “dimension”, which contain relatively rough and pregeometric information.
- Natural candidates may be spectral properties other than  $D_S(\sigma)$  of operators of Laplace type. However, they must have meaningful averages in the ensemble of geometries.
- Another cautionary remark: the geometries appearing in nonperturbative quantum gravity are *not* semiclassical (i.e. close to some  $g_{\mu\nu}$ ) on short distances, but at most after suitable coarse-graining (which is in itself a nontrivial affair).
- I hope to find inspiration for constructing other “observables” from the studies of fractal structures in other contexts, and from talking to the various experts present at this workshop!

# Causal Dynamical Triangulations - Summary & Outlook

CDT is a path integral formulation of gravity, which incorporates the dynamical and causal nature of geometry. It depends on a minimal number of assumptions and ingredients and has few free parameters. Its associated toolbox provides us with an “experimental lab” - a nonperturbative calculational handle on (near-)Planckian physics (c.f. lattice QCD).

- ➔ We have begun to make quantitative statements/predictions.
- ➔ We can in principle also test nonperturbative predictions from other fundamental theories containing gravity.
- ➔ Many nonperturbative lessons learned so far: relevance of metric signature in the path integral; tendency of geometric superpositions to degenerate; dynamical nature of “dimension”; emergence of classicality from quantum dynamics [not covered in today’s talk: crucial role of “entropy”; cure of the conformal divergence; role of “time” and “causality” as fundamental, and not emergent quantities]
- ➔ Hopefully we are seeing glimpses of an essentially unique quantum gravity theory; help us find good “observables” to understand it better!

## Where to learn more

- CDT light: “The self-organizing quantum universe”, by J. Ambjørn, J. Jurkiewicz, RL (Scientific American, July 2008)
- A nontechnical review in Contemp. Phys. 47 (2006) [arxiv: hep-th/0509010]
- recent reviews/lecture notes: arXiv 0906.3947, 1004.0352, 1007.2560, Physics Report to appear
- links to both review and popular science material can be found on my homepage <http://www.phys.uu.nl/~loll>



Quantum Gravity at the Planck Scale:

Getting a Handle on  
'Spacetime Foam'

Haifa,  
27 Jun 2011

**The End**