

Quantum dynamics for a triadic Cantor set spectrum

A. Soret[†], E. Gurevich [‡] and E. Akkermans[‡]

¹*Department of Physics, Technion Israel Institute of Technology, 32000 Haifa, Israel*

(Dated: June 25, 2015)

We study the dynamics of a wave packet for a triadic Cantor spectrum K . The scaling symmetry of the spectrum translates into the characteristic log-periodic feature of relevant physical quantities. Using Mellin transform, we derive the analytical expression of the time-averaged return probability, which is universal for any system with a Cantor spectrum. This result is supported by numerical results obtained recently for polaritons in a Fibonacci potential. We give a more thorough analysis in the case of the tight-binding model, in which case the dynamics of the system depend essentially on the Fourier transform on a Cantor set of monomes x^k , namely : $h_k(t) = \int_K e^{-ixt} x^k d\mu$, $k \in \mathbb{Z}$, which carry the fingerprint of the spectral fractal feature.

I. INTRODUCTION

II. TIME-AVERAGED RETURN PROBABILITY

A. Analytical expression

Note that $C(t)$ verifies asymptotically the functional equation of a scaling function :

$$C(t) \sim \frac{1}{2}C(t/3)$$

B. Numerical results for the Fibonacci cavity

III. GENERALIZATION TO OTHER QUANTITIES

Time-averaging is equivalent to taking the wavelet transform of the spectrum, with we have shown to display a characteristic behaviour. Therefore, it is expected that other time-averaged dynamical quantities would also display the same behaviour. To illustrate this :

- participation ratio (Fibonacci potential and triadic Cantor set spectrum)
- RMS (triadic Cantor set spectrum).

However, only qualitative argument : more information about the eigenfunctions is required to study rigorously the cases of the participation ratio and of the RMS.

IV. TIGHT-BINDING MODEL

Allows us to obtain more information on the eigenfunctions themselves, in order to study non-spectral quantity.

The dynamics of the system depends essentially on the Fourier transforms of monomes :

$$h_k(t) = \int_K e^{-ixt} x^k d\mu(x)$$

- These functions are not scaling functions, but display a structure, which translates in some dynamical quantities
- Application to the RMS
- taking the time-average

V. CONCLUSION

VI. REFERENCES