

N2 Fan.

Try: Perturbation theory in

J_1 and J_2

$$\delta H_1 = J_1 (\sigma_1^x)^i (\sigma_1^x)^{i+1}$$

$$\delta H_2 = J_2 (\sigma_2^y)^i (\sigma_2^y)^{i+1}$$

To find $|GS\rangle$

$$(\delta H_2)(\delta H_1) |GS\rangle = \bar{E}_0 |GS\rangle$$

$\bar{E}_0 < 0$

We choose a basis for the entangled states (determined by large $|J_2|$).

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{pmatrix} \Leftrightarrow \begin{matrix} \chi_1 |++\rangle_i^i \otimes |++\rangle_{i+1}^{i+1} \\ \chi_2 |++\rangle_i^i \otimes |--\rangle_{i+1}^{i+1} \\ \chi_3 |--\rangle_i^i \otimes |++\rangle_{i+1}^{i+1} \\ \chi_4 |--\rangle_i^i \otimes |--\rangle_{i+1}^{i+1} \end{matrix}$$

Next we try to find the matrix representation

$$\text{for } (\delta H_2 \delta H_1) = M.$$

Following the same procedures as we discussed today (in Karyn's note Page 3-4), we can verify:

$$(\delta H_2 \delta H_1) |++\rangle_i^i \otimes |++\rangle_{i+1}^{i+1} = (-J_1 J_2) |--\rangle_i^i \otimes |--\rangle_{i+1}^{i+1}$$

$$(\delta H_2)(\delta H_1) |++\rangle_i^i \otimes |--\rangle_{i+1}^{i+1} = (+J_1 J_2) |--\rangle_i^i \otimes |++\rangle_{i+1}^{i+1}$$

$$(\delta H_2 \delta H_1) |--\rangle_i^i \otimes |++\rangle_{i+1}^{i+1} = (+J_1 J_2) |++\rangle_i^i \otimes |--\rangle_{i+1}^{i+1}$$

$$(\delta H_2 \delta H_1) |--\rangle_i^i \otimes |--\rangle_{i+1}^{i+1} = (-J_1 J_2) |++\rangle_i^i \otimes |++\rangle_{i+1}^{i+1}$$

where the index s is ignored.

Thus

$$\delta H_2 \delta H_1 = M = \begin{pmatrix} 0 & 0 & 0 & -J_1 J_2 \\ 0 & 0 & J_1 J_2 & 0 \\ 0 & J_1 J_2 & 0 & 0 \\ -J_1 J_2 & 0 & 0 & 0 \end{pmatrix}$$

with the eigenvalue λ

$$(\lambda^2 + (J_1 J_2)^2)(\lambda^2 - (J_1 J_2)^2) = 0.$$

we get the ground state energy

$$\lambda = \bar{E}_0 = -J_1 J_2 < 0.$$

and eigenvectors satisfying

$$\begin{cases} \chi_1 - \chi_4 = 0 \\ \chi_2 + \chi_3 = 0 \end{cases}$$

So the ground states in this case:

$$|GS\rangle_{\alpha\beta} = \alpha \left(|++\rangle_i^i \otimes |++\rangle_{i+1}^{i+1} + |--\rangle_i^i \otimes |--\rangle_{i+1}^{i+1} \right) + \beta \left(|++\rangle_i^i \otimes |--\rangle_{i+1}^{i+1} - |--\rangle_i^i \otimes |++\rangle_{i+1}^{i+1} \right)$$

with $2(|\alpha|^2 + |\beta|^2) = 1$