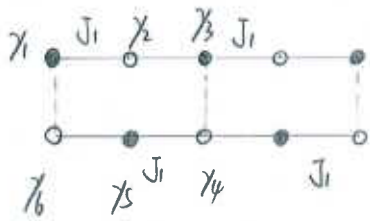


N 1

(18/11/2016)



$$J_2 = 0$$

$$J_3 = 0$$

free γ_i 's Majoranas 1 / per site.

How do we implement this \mathcal{H} ?

$$O_p = i \prod_{j=1}^b (\gamma_j)$$

$$H = -u \sum_p O_p, \quad (u > 0)$$

Take site 2 & 3 for example:

in MI report Eq.(52), the qubit on link (2-3) can be formed by unpaired γ_2, γ_3 ,

$$\begin{cases} f^+ = \frac{1}{2} (\gamma_2 + i\gamma_3) \\ f = \frac{1}{2} (\gamma_2 - i\gamma_3) \end{cases}$$

$$\Rightarrow \begin{cases} \gamma_2 = (f^+ + f) \\ \gamma_3 = -i(f^+ - f) \end{cases}$$

$$\begin{aligned} \gamma_2 \gamma_3 &= -i (f^+)^2 - f^2 - f^+ f + f f^+ \\ &= +i (2f^+ f - 1) \end{aligned}$$

$$= \begin{cases} +i & f^+ f = 1 \\ -i & f^+ f = 0 \end{cases}$$

$$\begin{aligned} \hat{U} &= \exp(i\pi f^+ f) = 1 - 2f^+ f \\ &= i\gamma_2 \gamma_3 \end{aligned}$$

$$\gamma_2^2 = \gamma_3^2 = 1$$

the total fermion parity operator:

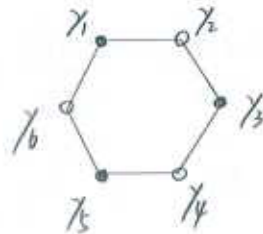
$$\begin{aligned} \text{Eq. (3)} \quad T &= i^{N/2} \prod_n \gamma_n \\ &= \prod_{j=2m} \pi (i\gamma_j \gamma_{j+1}), \end{aligned}$$

$$i\gamma_j \gamma_{j+1} = \begin{cases} -1 & f^+ f = 1 \\ 1 & f^+ f = 0 \end{cases}$$

depends on the occupancy of qubits.

2. How to realize (1) & (2)?

Acting only on 1 cell:



1° Apply J_3 on all sites:

ring exchange (6th-order perturbation)

$$\begin{aligned} \sigma_3^1 \sigma_3^2 \sigma_3^3 \sigma_3^4 \sigma_3^5 \sigma_3^6 &= \hat{U}_{23} \hat{U}_{45} \hat{U}_{61} \\ &= (-i\gamma_1 \gamma_2) (-i\gamma_3 \gamma_4) (-i\gamma_5 \gamma_6) \\ &= i \prod_{j=1}^6 \gamma_j \\ &= O_p = \hat{P}_{23} \hat{P}_{45} \hat{P}_{61} \end{aligned}$$

$i\gamma_2 \gamma_3$ can be fixed by changing sign of J_3 .

$$\begin{cases} t=0^- , J_3 > 0, & -i\gamma_2 \gamma_3 = -1, f^+ f = 0 \\ t=0^+ , J_3 < 0, & -i\gamma_2 \gamma_3 = +1, f^+ f = 1 \end{cases} \Rightarrow \text{change states of qubits.}$$

2° $\partial H = J_3 \sum \sigma_3^j \sigma_3^{j+1}$

3rd order: "ring exchange"

$$1st: \frac{\partial \hat{H}_{23}}{\partial J_B} = i\gamma_2\gamma_3$$

$$2nd: (\sigma_B^2 \sigma_B^3) (\sigma_B^4 \sigma_B^5) = (i\gamma_2\gamma_3)(i\gamma_4\gamma_5)$$

$$\hat{H}_{j,j+1} = \exp(i\pi f^{\dagger}f) J_B$$

$$\frac{\partial \hat{H}_{j,j+1}}{\partial J_B} = e^{i\pi f^{\dagger}f} = i\gamma_j\gamma_{j+1}$$

For the three link measurement:

$$\left\langle \frac{\partial \hat{H}_{12}}{\partial J_B} \right\rangle \cdot \left\langle \frac{\partial \hat{H}_{23}}{\partial J_B} \right\rangle \cdot \left\langle \frac{\partial \hat{H}_{34}}{\partial J_B} \right\rangle$$

$$\approx \left\langle \frac{\partial \hat{H}_{12}}{\partial J_B} \cdot \frac{\partial \hat{H}_{23}}{\partial J_B} \cdot \frac{\partial \hat{H}_{34}}{\partial J_B} \right\rangle$$

$$= \left\langle \hat{O}_P \right\rangle \quad \text{"Ring operator"}$$