
Basic Probability Revision

Exercise 1. Dices.

- 1) Consider a fair six faces dice. What is the probability to get a 4 ?
- 2) Determine the probability to get a number greater than 4.
- 3) Throw three fair dices. Calculate the probability for the sum to be greater than 4.
- 4) In average, how much will I get if I throw the dice once ?
- 5) If the dice has n faces, what is the average result of a throw ?
- 6) Estimate the standard deviation of one throw.

Exercise 2. Bernoulli and Binomial law.

We define a Bernoulli game with probability p , i.e. a random variable Y such that $\mathbb{P}(Y = 1) = p$ and $\mathbb{P}(Y = 0) = 1 - p$. The law of this random variable is often noted $\mathcal{B}(p)$. The sum of the repetition of this variable n times is a binomial game, noted X_n .¹

- 1) What is the probability to get k successes, i.e. calculate $\mathbb{P}(X_n = k)$? Estimate the expectancy of X_n .
- 2) What is the standard deviation of X_n ?
- 3) Give an asymptotic approximation of X_n .

Exercise 3. Ballot boxes and Polya urn models.

In this exercise, we consider an urn with r red ballots and n black ballots in it. We take two ballots whitout replacing the first one in the urn.

- 1) What is the probability for the 2 ballots to be red ?
- 2) Estimate the probability for the first to be red and the second black.
- 3) Let's modify the experiment : we get one ballot and note its colour. Then, we put d ballots of this kind plus this one in the urn. We repeat this process as long as we need a new step.

1. If Y_1, \dots, Y_n are n independent Bernoulli games of same probability p , X_n has the same distribution than $Y_1 + \dots + Y_n$.

- a) *What is the probability for the second ballot to be black?*
- b) *What is the probability for the first ballot to be black if we know the second is black?*

Exercise 4. *Convergence theorems.*

Let $(X_i)_{1 \leq i \leq n}$ be independent identically distributed random variables. Let $(S_n)_{n \geq 1}$ be defined by $S_n = \frac{1}{n} \sum_{i=1}^n X_i$.

- 1) *Study the convergence of S_n and give an asymptotic distribution.*
- 2) *What is the rate of convergence?*