

Fourier Optics

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We studied the imaging properties of a simple $4f$ system, in the setting of Fourier Optics. We imaged gratings of varying frequency under illumination of various coherence. We observed the image both in spatial and frequency domain; enabling us to determine the transfer function of the system, as well as effects like ringing, aliasing (from the digital camera used), or defocus effects like the Talbot frequency doubling.

Introduction

The purpose of Fourier optics is to describe an optical system with the tools of the Fourier theory, and more precisely in terms of its effects over the frequency contents of an image.

The behaviour of the system is wholly different between conditions of coherent and incoherent imaging, and the first two parts of this reports will present those behaviours, as well as the measured transfer function for incoherent illumination. The last part will be a discussion of the behaviour of the system under conditions of mixed coherence and defocus, and a description of the effects due to the use of a digital camera.

Set-up

The optical system under study consisted in two identical lenses, sharing a common focal point where was placed the aperture stop. This is commonly called a $4f$ system (since the distance between the object and the image will be four times the common focal length). Here the focal length was about 9.0cm .

The light source was a red led, of power controlled by voltage. The effective spatial extension of the source was controlled by an iris, modulating the coherence of the illumination, and a lens is used to provide a uniform (Kohler) illumination.

The object used to analyse the system is a grating (gratings of several frequencies were available), placed at the focus of the first lens. A CCD camera is then placed at the focus of the second lens, recording the output of the system. It is then processed to display both the image and its spectrum (Fourier transform), analysed by a provided Matlab script.

The set-up is represented in the figure 1, taken from the lab script.

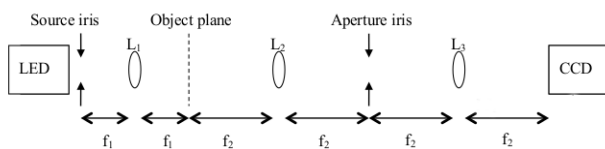


Fig. 1. Schematic of the $4f$ system

1. First observations

In the case where the aperture iris is completely open, and when the system is perfectly focused, the image is exactly the intensity distribution at the first focal plane of the first lens. (That is, here, a grating).

The Fourier transforming properties of a thin lens tell us that we are to observe in the second focal plane of the first lens the Fourier spectrum of the intensity distribution in the first focal plane; and indeed we putting a piece of paper instead of the aperture stop, we observed a line of points, which is the Fourier transform of a grating. (Cf. also subsection 5.2.2 of [1]).

If the source iris is small enough, then the illumination will be coherent. That is, the system will be linear in amplitude: at one point, two incoming waves would sum up and the observed intensity will present interference effects. In that case, the Fourier Optics theory tells us that the coherent transfer function is simply a scaled version of the pupil function (here, just a disk). (cf. also Section 6.2 of [1]). The hard-edge low-pass nature of this filter gives rise to the appearance of ringing artefacts. This is exactly the Gibbs phenomenon, where truncated Fourier series give rise to over-oscillations.

The Fourier transform of a brick-wall filter is an oscillating function; here the image is convolved with an Airy function. This causes oscillatory pattern to appear near high-contrast boundaries in the image. This is clearly illustrated in Fig. 2.

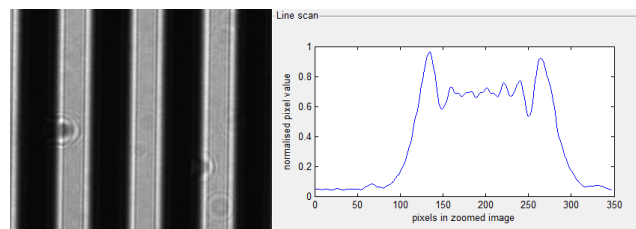


Fig. 2. Image and line scan for the ringing effect

2. Incoherent illumination

When the size of the source is big enough, the illumination becomes incoherent, and the system is linear in term of intensity. In that case, it may be shown that the optical transfer function of the system will

be the autocorrelation of the coherent transfer function. As, in our case, the latter is simply the scaled aperture stop, the incoherent transfer function will be $\frac{2}{\pi} \left(\arccos \left(\frac{r}{2r_0} \right) - \frac{r}{2r_0} \sqrt{1 - \left(\frac{r}{2r_0} \right)^2} \right)$ for $r \leq r_0$ where r_0 is the frequency cut-off of the coherent system (cf. for reference Section 6.3 of [1]). There are two important differences with the coherent case: first, the frequency cut-off is twice as much, and secondly, the filter is a soft cut-off, without noticeable ringing effects.

For three values of the diameter of the aperture stop (fully open, 5cm and 10cm), we measured the intensity of the spectral components to deduce the form of the incoherent transfer function. To be able to measure every peak, we used the grating of highest frequency at our disposal, which gave a bigger inter-peak separation than lower frequency gratings. A grating can be idealized in Fourier space as a sum of Dirac peaks, at *odd*-integer harmonics of the frequency of the grating; each peak having a coefficient of $\frac{1}{n}$ where n is the harmonic order. Thus, we measured the intensity of every two peaks, multiplied them by n and plotted it as a function of frequency. The results are presented in Fig. 4. We can see that the overall shape does not really match the ideal transfer function; it is well possible that this is an effect of an imperfect focusing of the whole system (cf. [1], Figure 6.11b p.150). The fundamental grating frequency was measured to be 8.43mm^{-1} .

3. Beyond ideal: mixed coherence, defocus, aliasing

When changing from coherent to incoherent illumination, we observed that, for a given aperture, additional harmonic components would appear; this illustrate well how the system changes from a hard-edge low-pass filter of cut-off frequency r_0 to a soft-edge low-pass of cut-off $2r_0$.

We also observed the effect of defocus over our system; when shifting the position of the plate, moving it backwards, we observed first a doubling of the frequency on the image of the grating, then an image of the grating but with reversed contrast. This is linked with the Talbot effect : the near-field diffraction pattern of the grating is taken as object in the first focal plane of the first lens. (For the Talbot effect, cf. Section 4.5.2 of [1])

Finally, as we are using a digital system to observe the image, the said image is sampled by the CCD pixel array. As such it is subject to aliasing : sampling can be understood as a multiplication by a Dirac Sha distribution (comb); which in frequency gives a convolution of the object spectrum by a comb of inverse frequency. If the sampling frequency is smaller than twice the higher spatial frequency contained in the image, then the spectrum will overlap. This is shown in Fig. 3, where the line of frequency components wraps around the 2d plane

and produces parallel lines of points. This effects disappeared with the closing of the aperture stop, showing clearly its function of filtering in Fourier space.

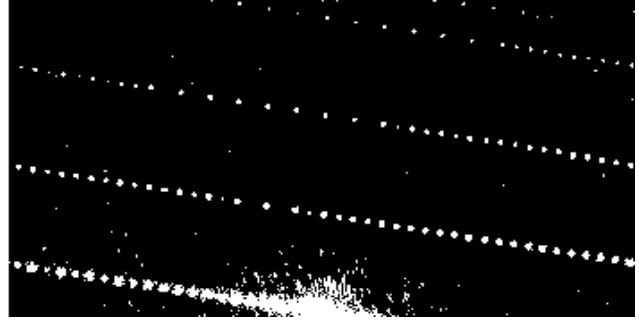


Fig. 3. Aliasing (upper half of the spectrum, processed to make the wrapped spectrum visible)

Conclusion

This 4f system provides considerable insight in the theory of Fourier optics, as it links to significant results: one being that a lens realises the Fourier transform of its front focal plane in its back focal plane, and the other being that a general imaging system realises the multiplication of the Fourier spectrum of the object by its transfer function. Here it is highlighted as the transfer function comes mostly from the aperture stop, placed precisely at the common focal plane of the two lenses, realizing directly the frequency filtering process for the coherent case, and showing the difference in behaviour of the incoherent case.

References

1. J.W. Goodman, "Introduction to Fourier Optics", third edition, Roberts & Company (2005).
2. Lab script "Fourier Optics - Investigating the performance of a simple imaging system", Imperial College (2005).

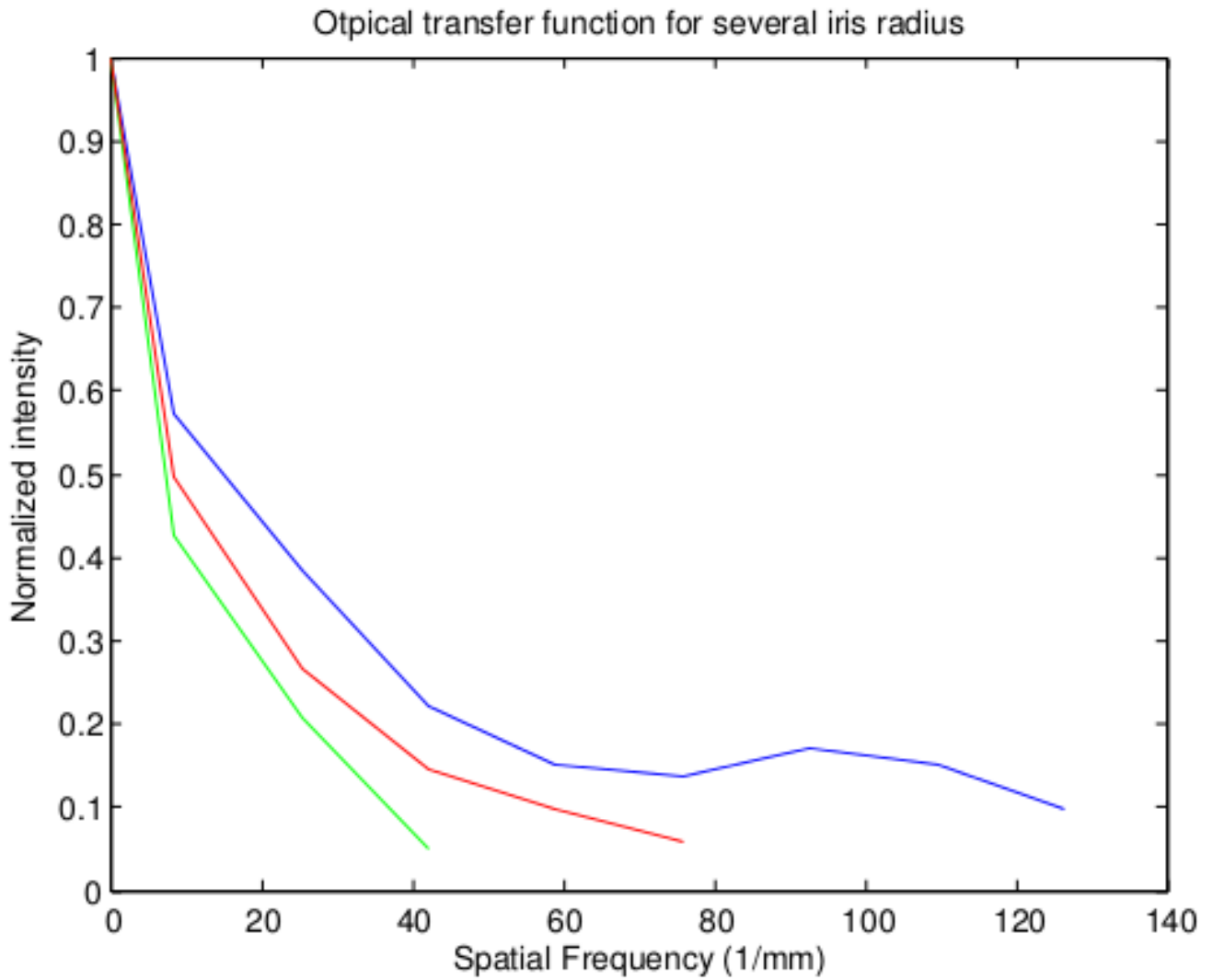


Fig. 4. Incoherent transfer function. Diameter: green: 5mm, red: 10mm, blue : wide open