

The atomic nucleus : A natural laboratory of complexity

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Definitions

The atomic nucleus

Complexity

Description(s)

Generalities

Many-body techniques

State-of-the-art descriptions

Problems

Beyond Mean Field

Approaches

Conclusions

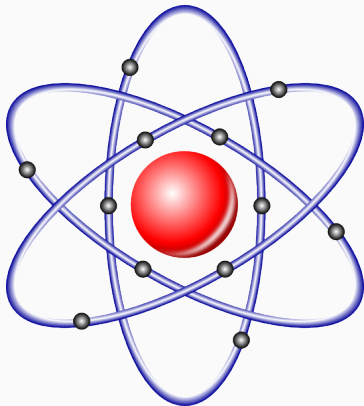
We are still there

Outlook

Definitions

Is it a nucleus ?

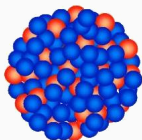
A **building** block of matter



- Small and dense region at the center of the Atom
- Inferred by Rutherford
- "Made" of protons and neutrons

Is it a nucleus ?

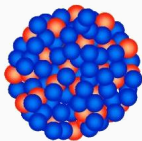
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A **building** block of matter



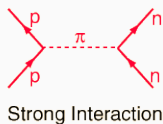
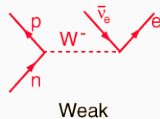
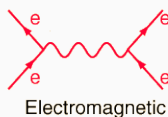
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What are the correct d.o.f to describe it ?

What is a nucleus ?

A quantum-many-body system:

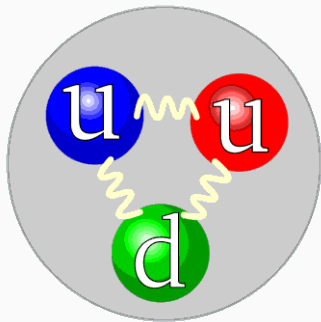
- Sensitive to 3 fundamental interactions (EM,W,S)
- Composed of non-elementary fermions
- Strongly correlated system of finite size



What is a nucleus ?

A quantum-many-body system:

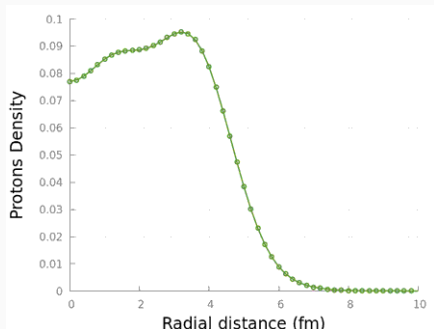
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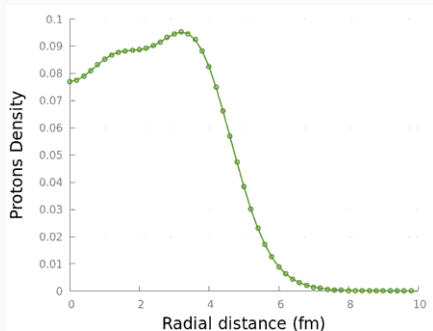
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A quantum-many-body system:

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The nucleus is a complex system

Complexity – A challenging description

Consequences:

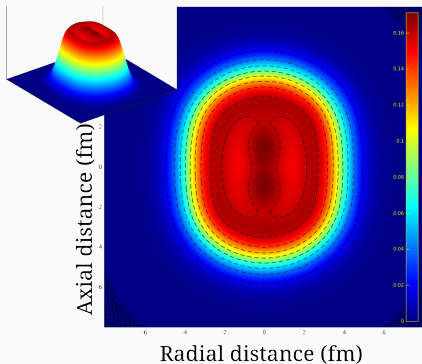
- No systematic analytical treatment of the problem.
- A wide variety of phenomena.

Complexity – A challenging description

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Deformation:



Complexity – A challenging description

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Clustering:

Complexity – A challenging description

Consequences:

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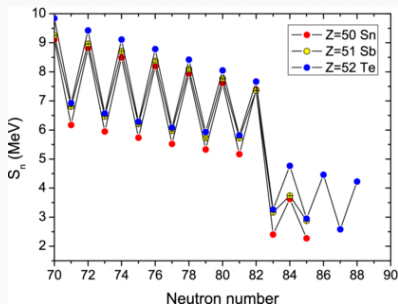
Superfluidity:

What can one measure ?

Some observables

Ground-State:

- Energy (Separation)
- Radii

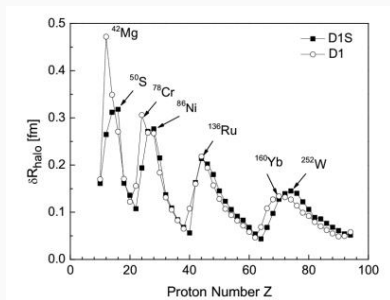


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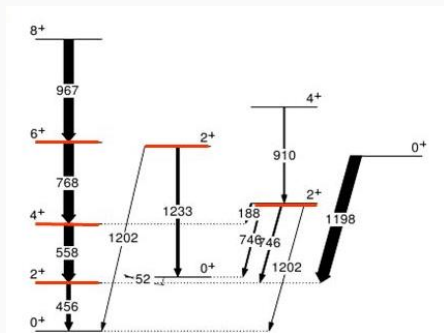


What can one measure ?

Some observables

Excited-states:

- Energies (Spectroscopy)
- Electric moments

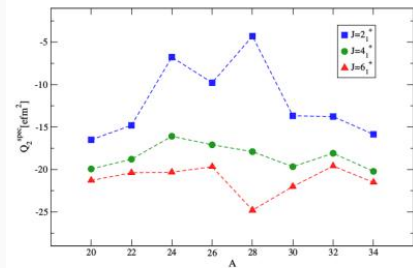


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Some observables

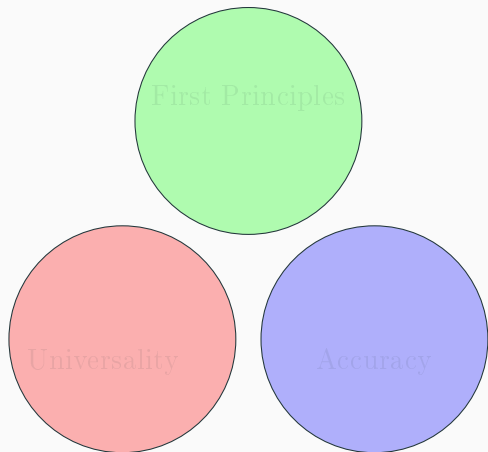
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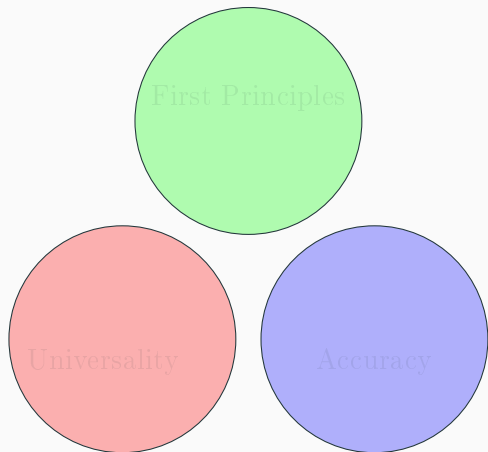
Description(s)

Nuclear "Philosophy" – A complicated compromise



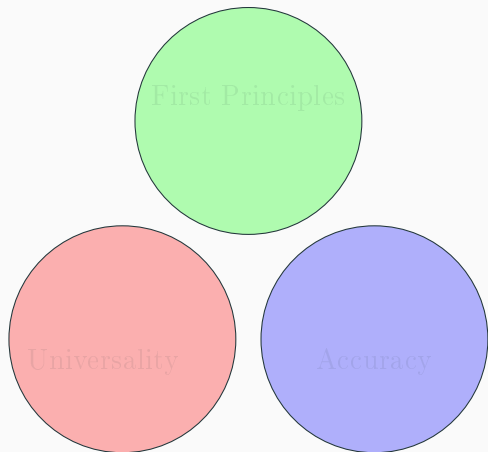
- A scaling problem
- Treatment of the many-body problem
- Description of the interaction

Nuclear "Philosophy" – A complicated compromise



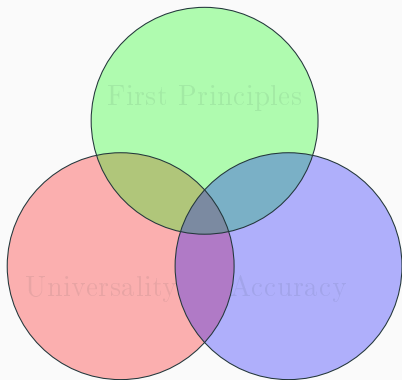
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We need a compromise !

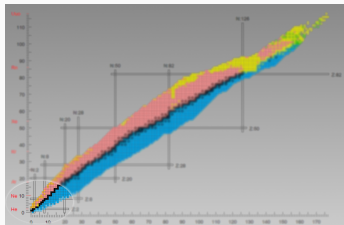
The quantum Many-body problem

- Exact
- Ab-initio
- EDF



Summary: Explicit treatment of the total wave-function

- $\hat{H}\Psi = E\Psi$
- Very accurate with a true interaction.
- Extremely costly and heavy (Power-law scaling)



The quantum Many-body problem

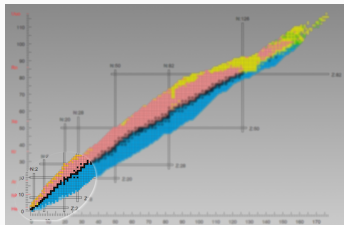
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$$|\Psi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle + c_3|\Phi_3\rangle + \dots + c_n|\Phi_n\rangle + c_{n+1}|\Phi_{n+1}\rangle + \dots$$

1-particle 1-hole 2-particle 2-hole

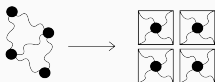
Summary: Explicit treatment of truncated total wave-function

- $\Psi = \sum_I c_I \Phi_I$
- Predictive with an effective interaction
- Very costly and heavy (Combinatorial scaling)



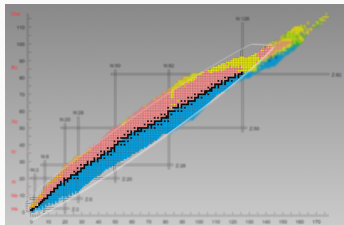
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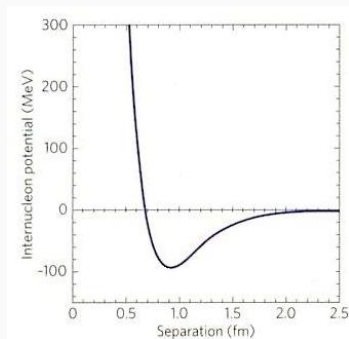
Summary: Mean-field like treatment of the wave-function

- $[\hat{H}, \hat{\rho}] = 0$
- Almost universal, but uses a phenomenological interaction
- Quite computationally easy (Polynomial scaling)



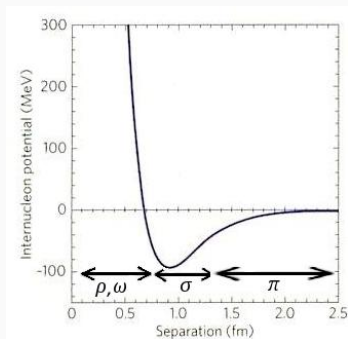
Energy Functionals – A Phenomenological approach

An example Relativistic Mean Field Theory (RMF)



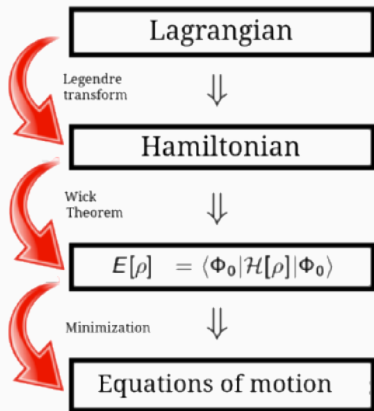
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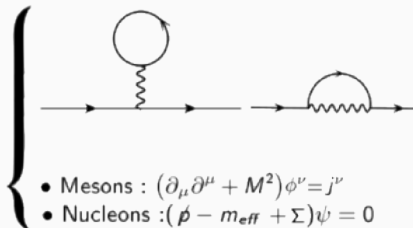
Interpretation in term of mesons exchange

A brief summary



$$\mathcal{L}_{int} = g_\sigma \bar{\psi} \sigma \psi + g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi + g_\rho \bar{\psi} \gamma_\mu \rho^\mu \cdot \vec{\tau} \psi + g_\pi \bar{\psi} \gamma_5 \vec{\pi} \cdot \vec{\tau} \psi$$

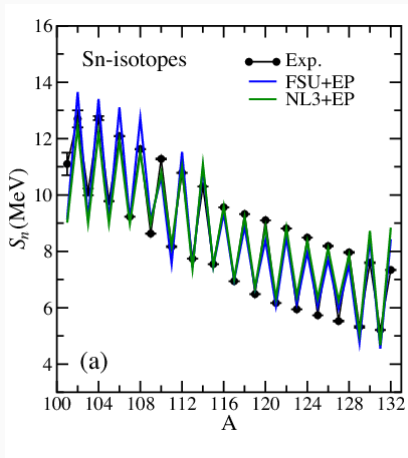
$$\mathcal{H} = \hat{T}_{i,j} + \hat{V}_{eff}$$



State-of-the-art descriptions

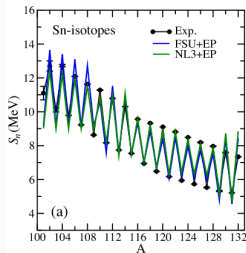
Correlations

Some "observable" problems



Correlations

Some "observable" problems

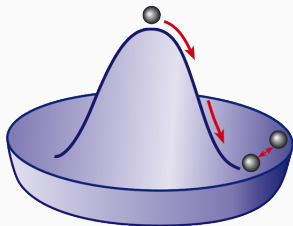


Odd-even staggering \Rightarrow Pairing correlations



Correlations

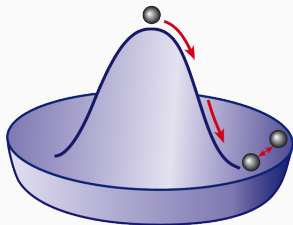
Symmetry breaking:



- Capture additional correlations
- For any sym. group. ($U(1), O(3), \text{etc...}$)

Correlations

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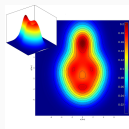


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And projection:

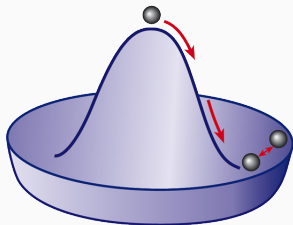
- Integrate over the configurations

$$\bullet \hat{P}^A = \frac{1}{2\pi} \int_0^{2\pi} e^{i\psi(\hat{N}-A)} d\psi$$



Correlations

Symmetry breaking:

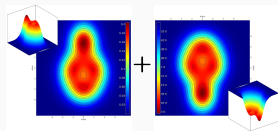


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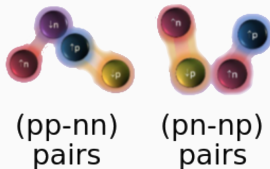
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Pairing correlations

Kind of pairs



BCS/Bogoliubov

$$\underbrace{|\Psi_0\rangle}_{\text{Sym. Broken}} = \prod_k \left(u_k + v_k \underbrace{a_{k,\uparrow}^\dagger a_{-k,\downarrow}^\dagger}_{\text{Pair creation}} \right) \underbrace{|0\rangle}_{\text{Sym.}}$$

Projection

$$\hat{P}^A |\Psi_0\rangle = \underbrace{\frac{1}{N!} (\Gamma_\tau^\dagger)^N |0\rangle}_{\text{Sym. Restored !}}$$

Spatial properties of pairing correlations

How is a pair localized ?

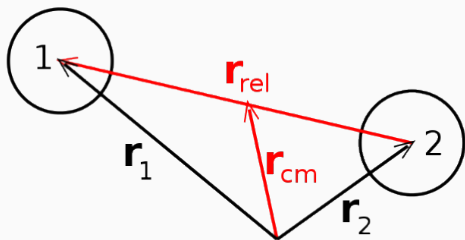
How is a pair localized ?

Motivations

- Better understanding of pairing impact on the structural properties
- Are the pairs a good d.o.f ?
- Important for experimental studies¹

¹R. Subedi et al., Science 320 (2008) 1476

Reparametrization



$$(\vec{r}_1, \vec{r}_2) \rightarrow (\vec{R}_{cm}, \vec{r}_{rel})$$

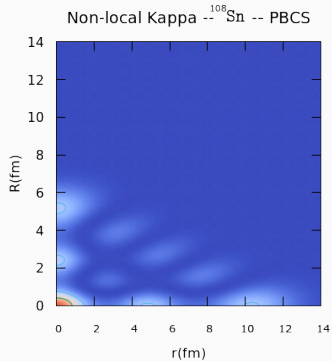
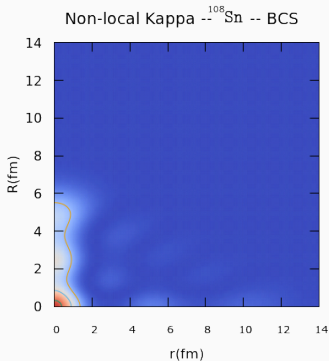
$$\hat{O}(r_1, r_2) \rightarrow \boxed{\text{Talmi transformation}} \rightarrow \hat{O}(r_{cm}, r_{rel})$$

High numerical cost !

Insight on the structure

Two useful variables:

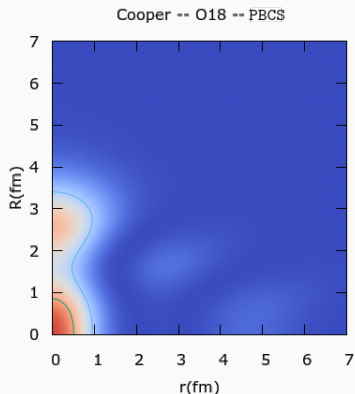
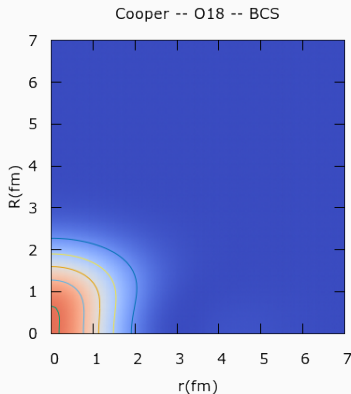
$$\kappa(r_1, r_2) = \sum_k u_k v_k \psi_k(r_1) \gamma^0 \psi_{\bar{k}}(r_2)$$



Insight on the structure

Two useful variables:

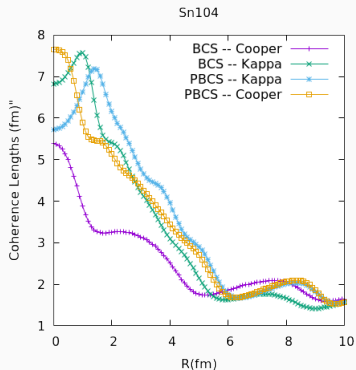
$$\Psi(r_1, r_2) = \sum_k \frac{u_k}{v_k} \psi_k(r_1) \gamma^0 \psi_{\bar{k}}(r_2)$$



Coherence Lengths

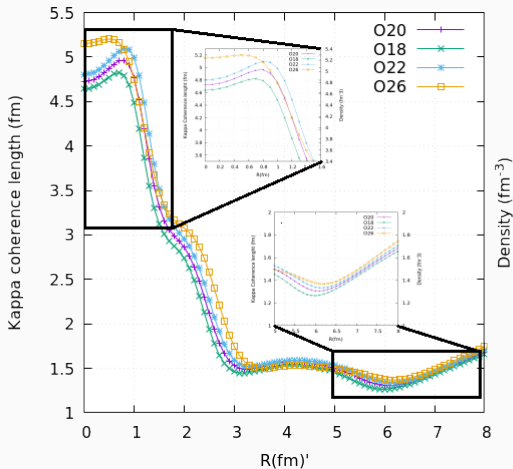
Defined as

$$\sqrt{\frac{\int dr_{\text{rel}} r_{\text{rel}}^4 \hat{O}(r_{\text{cm}}, r_{\text{rel}})}{\int dr_{\text{rel}} r_{\text{rel}}^2 \hat{O}(r_{\text{cm}}, r_{\text{rel}})}}$$



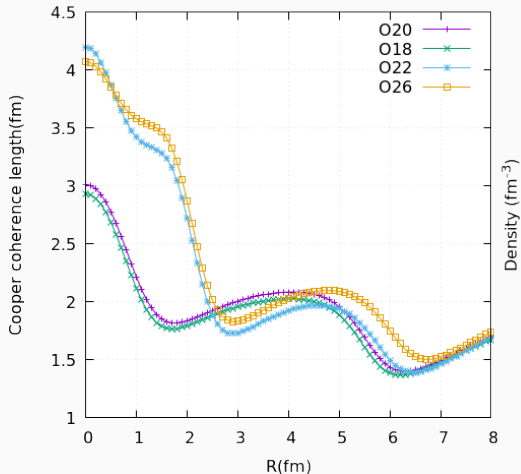
Results and applications

Isotopic study



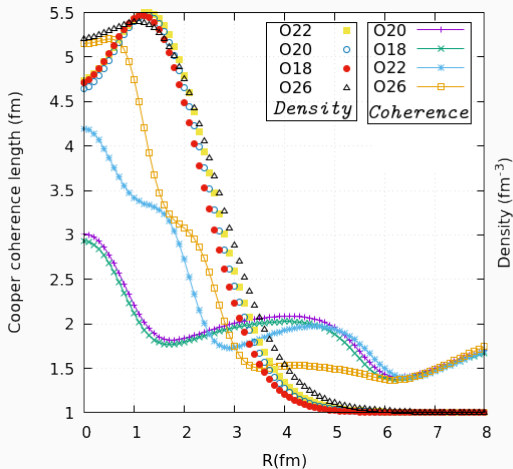
Results and applications

Isotopic study



Results and applications

Isotopic study



Conclusions

New powerful tools to study spatial correlations

- Accurate reproduction of physical properties (Pauli blocking, saturation ...)
- A step toward a better understanding of experimental observables.
- Impact of restoration techniques on nuclear structure.

Nuclear physics is an active research field !

Major intrinsic open subjects:

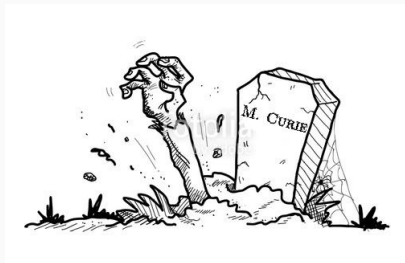
- Theoretical link between QCD and N-N Interaction
- Systematic and simultaneous restorations
- New (non-spurious) many-body techniques.
- Precise study of correlations

A dynamic field

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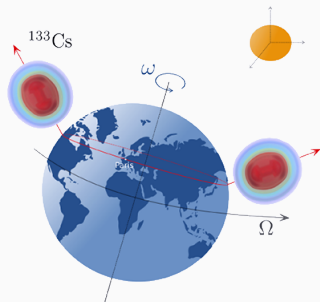


Detection of Majorana neutrinos

- Precise study of double-beta decays
- Dependence on nuclear reactions knowledge



Lorentz-symmetry breaking²



- Very strong dependence on nuclear structure knowledge
- Major cosmological impact

²H.Pians-Le Bars, C. Guerlin, **R-D.L**, J-P. Ebran, Q.G. Baily, S.Bize, E.Khan, P.Wolf **Phys.Rev.D** **95**,075026

Thank you !

