

Open boundary conditions for SPH representing free surface flows

A big (?) step to handle SPH-Boussinesq coupling

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Outline

- 1 Introduction & Context
- 2 First attempt: imposing velocity/pressure
 - Description
 - Examples
- 3 Second attempt: boundary for SWE
 - Description
 - Example: steady flow over a bump
- 4 Conclusion

Introduction

Waves model (Boussinesq, Saint-Venant)

- Unable to represent complex free surface (multi-connected domains)
- Can represent sloshing with damping [Benoit 02, Yu 99]
- Studies shows the necessity of more physical models [Duthyk 10]

Sloshing representation (VOF, SPH)

- Waves damping (can be handled by *ad-hoc* treatment)
- Computational coast (3D computations un-reachable)

Coupling is a natural choice



Wave propagation

Boussinesq model – results



Breaking wave

Using SPH model

- Usual boundary condition: solid wavemaker
- We want to avoid it and implement Open Boundary Condition



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Open boundary condition

Using a buffer zone

- To complete particle kernels
- Where we can impose pressure/velocity
- Handle deleting/creating particles

Problems

- No-free surface at the boundary
- If other quantities should be imposed?



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- If other quantities should be imposed? **Riemman invariant**




Open boundary condition



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Open boundary condition based on shallow water equations

[Vacondio *et al* 10]

Shallow water equations

- $D_t \rho h = -\rho h \nabla \cdot \bar{\mathbf{v}}$
- $D_t \bar{\mathbf{v}} = -h \nabla \rho h + g (\nabla b + \mathbf{S}_f)$

Open boundary condition based on shallow water equations

Shallow water equation boundary conditions

- Inflow $\bar{\mathbf{v}} \cdot \mathbf{n}_{\text{out}} < 0$
- Outflow $\bar{\mathbf{v}} \cdot \mathbf{n}_{\text{out}} > 0$
- Subcritical $\|\bar{\mathbf{v}}\| < \sqrt{gh}$
- Supercritical $\|\bar{\mathbf{v}}\| > \sqrt{gh}$

Use Riemann invariants at OB according to local Froude number

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Subcritical outflow:

- h is imposed
- $\bar{v}_1 = \bar{v}_{i,1} + 2\sqrt{g} \left(\sqrt{h_i} - \sqrt{h} \right)$
- $\bar{v}_3 = 0$

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Subcritical inflow:

- $\bar{\mathbf{v}}$ is imposed
- $$h = \left[\frac{1}{2\sqrt{g}} (\bar{v}_{i,1} - \bar{v}_1) + \sqrt{h_i} \right]^2$$

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Supercritical outflow:

- $\bar{v}_1 = \bar{v}_{i,1}$
- $h = h_i$

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Supercritical inflow:

- $\bar{\mathbf{v}}$ is imposed
- h is imposed

OBC based on shallow water equations

Compute SWE unknowns with SPH

Compute water depth using at $\mathbf{x} = (x_1, x_3)$:

$$\rho(x_1, x_3) = \sum_j \rho_j W(\mathbf{x} - \mathbf{x}_j) \frac{m_j}{\rho_j}$$

Water for $x_3 \in [b, h]$ with $\rho(x_1, x_3) > \alpha \rho_w$

Here $\alpha = 0.5$

Mean velocity computed as:

$$\bar{v}_1(x_1) = \frac{1}{h-b} \int_b^h v_1(x_1, x_3) dx_3$$

OBC based on shallow water equations

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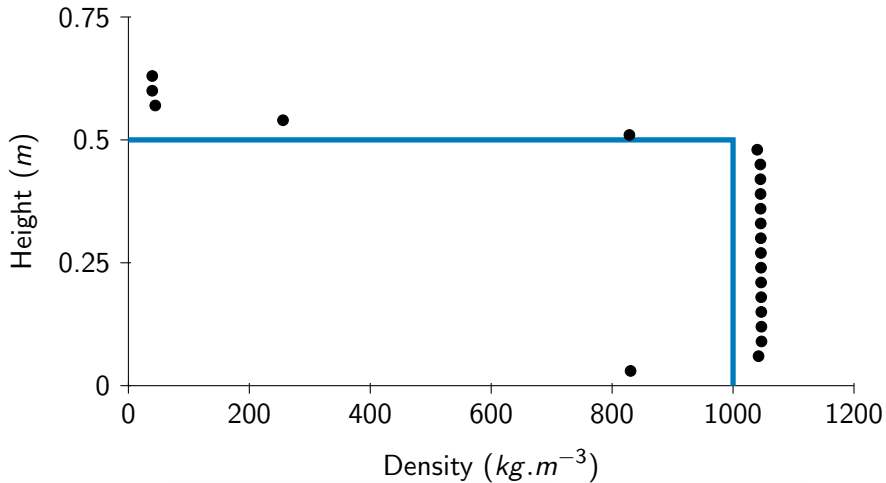
Here $\alpha = 0.5$

Mean velocity computed as:

$$\bar{v}_1(x_1) = \frac{1}{h-b} \int_b^h v_1(x_1, x_3) dx_3 \simeq \frac{1}{h-b} \sum_{k=1}^{\frac{h-b}{\Delta x_3}} v_1(x_1, k \Delta x_3) \Delta x_3$$

OBC based on shallow water equations

Compute SWE unknowns with SPH





OBC based on shallow water equations

Remarks

Initialize buffer zone

- Generate initial state using normal SphysicGEN
- OBC localization and default state in `obc.dat`
- Create buffer particle
- Delete out of bounds particle

Buffer zone behaviour

- Values imposed (for kernel completeness)
- Delete fluid particle entering buffer zone

Delete and create particles

- Deleted particles stored in *trash*
- Generate a particle using *trash* if possible



Water over a bump

Problem description

Problem used in [Vacondio *et al*, 10] to validate SPH-SWE

- Fluid domain $x_1 \in [0m, 10m]$
- Fluid bottom

$$b(x_1) = \begin{cases} b_0 \left[1 - \frac{(x_1 - 5)^2}{4} \right] & \text{if } x_1 \in [3m, 8m] \\ 0 & \text{elsewhere} \end{cases}$$

with $b_0 = 20cm$

- Analytical solution
- Subcritical or supercritical case



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Water over a bump

Large overview



Water over a bump

Zoom on inlet



Water over a bump

Zoom on outlet



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Conclusions and Outlooks

Conclusion:

- Open Boundary Condition based on Shallow Water Equations
- Use of Riemann invariants

Problems:

- If the flow does not verify SWE hypothesis? (open question)
- Automatic boundary condition switching (easy)
- Accurate computation of water depth (less easy)

Outlook

- Implementing SPH component
- Coupling with Boussinesq solver



Conclusions and Outlooks

Thank you for attention