


~~work~~

① $\begin{cases} x = 3 \sin t \\ y = \frac{3}{2\sqrt{2}} \sin t \cos t \end{cases}$ $\mathcal{D}_f = \mathbb{R}$ période 2π

$x(-t) = -x(t)$ x est impair
 $y(-t) = -y(t)$ y est impair

$\pi \in \mathcal{H}$ $\text{sym} / 0$



Étude sur $[0; \pi]$ + sym / 0. On récupère les pts pour $t \in [-\pi, 0]$
On a tous les points avec la période.

• $x'(t) = 3 \cos t$ $x'(t) = 0 \Leftrightarrow t = \frac{\pi}{2} + k\pi$ entre 0 et π : en $\frac{\pi}{2}$

• $y'(t) = \frac{3}{2\sqrt{2}} (\cos^2 t - \sin^2 t) = \frac{3}{2\sqrt{2}} (1 - 2\sin^2 t) = \frac{3}{2\sqrt{2}} \cos(2t)$

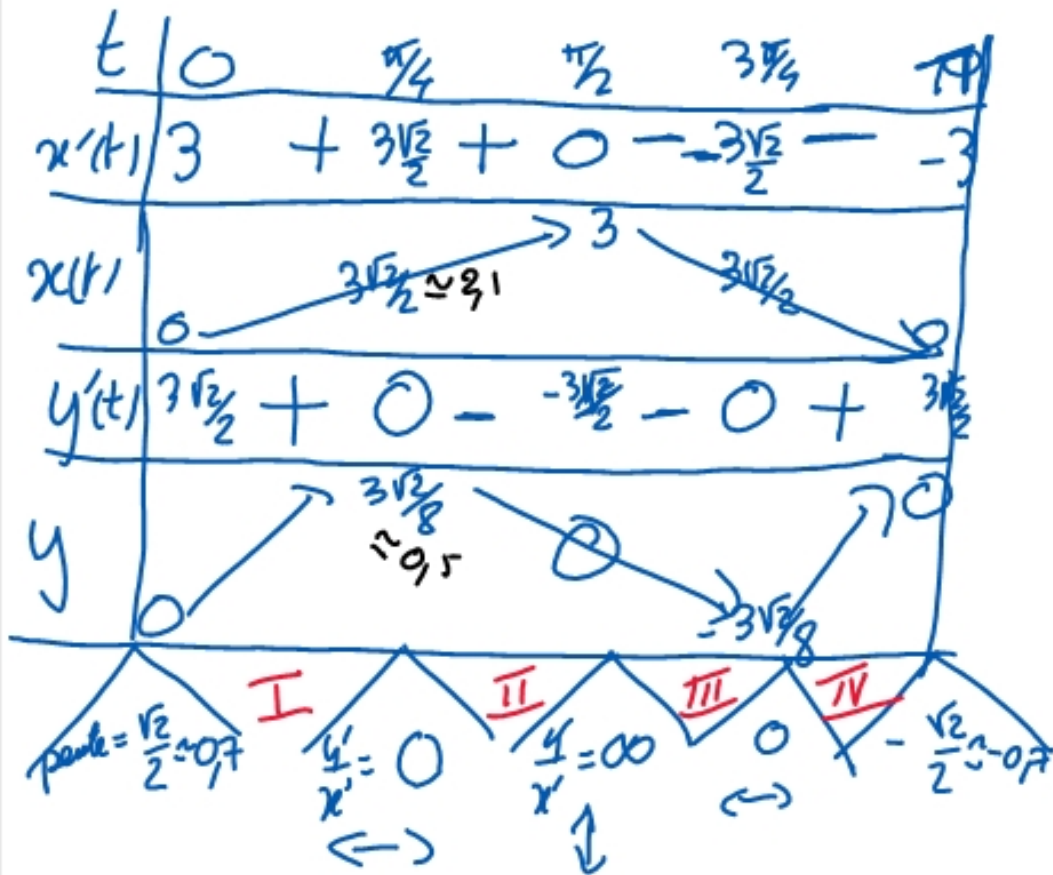
$y'(t) = 0 \Leftrightarrow \cos(2t) = 0 \Leftrightarrow 2t = \frac{\pi}{2} + k\pi \Leftrightarrow t = \frac{\pi}{4} + \frac{k\pi}{2}$ $\frac{\pi}{4}$ et $\frac{3\pi}{4}$

$$x(t) = 3 \sin t \quad x' = 3 \cos t$$

$$y(t) = \frac{3}{2\sqrt{2}} \sin 2t \quad y' = \frac{3}{\sqrt{2}} \cos(2t)$$

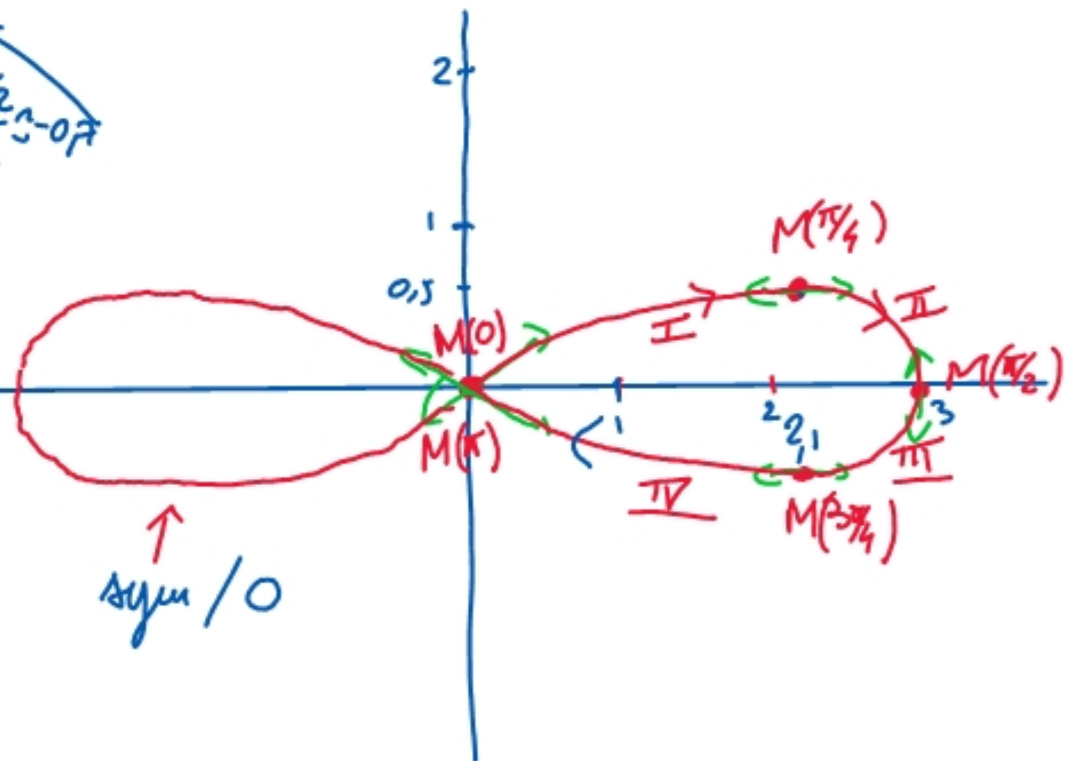
$$y\left(\frac{\pi}{4}\right) = \frac{3}{2\sqrt{2}} \frac{\sqrt{2}}{2} \frac{\sqrt{2}}{2} = \frac{3 \cdot 2}{8\sqrt{2}} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

pente en un point $\frac{y'}{x'}$



longueur: $L = 4 \int_{t=0}^{t=\frac{\pi}{2}} \sqrt{x'(t)^2 + y'(t)^2} dt$

$$L = 4 \int_0^{\frac{\pi}{2}} \sqrt{9\cos^2 t + \frac{9}{8}(\cos 2t - 1)^2} dt$$



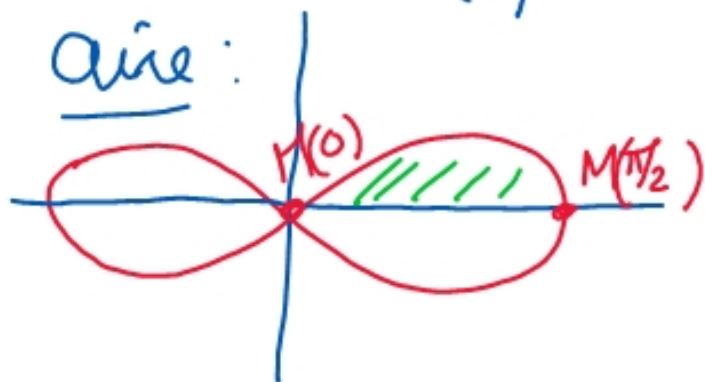
$$L = 4 \int_0^{\pi/2} 3 \sqrt{\frac{8\cos^4 t + (2\cos^2 t - 1)^2}{8}} dt = \frac{4 \cdot 3}{\sqrt{8}} \int_0^{\pi/2} \sqrt{8\cos^4 t + 4\cos^2 t - 4\cos^2 t + 1} dt$$

$$L = \frac{4 \cdot 3}{2\sqrt{2}} \int_0^{\pi/2} \sqrt{4\cos^4 t + 4\cos^2 t + 1} dt = \frac{6}{\sqrt{2}} \int_0^{\pi/2} \sqrt{(2\cos^2 t + 1)^2} dt = \frac{6\sqrt{2}}{2} \int_0^{\pi/2} (2\cos^2 t + 1) dt$$

$$L = 3\sqrt{2} \int_0^{\pi/2} (\underbrace{2\cos^2 t}_{\cos(2t)} + 1) dt = 3\sqrt{2} \int_0^{\pi/2} (\cos(2t) + 1) dt$$

$$= 3\sqrt{2} \left[\frac{\sin(2t)}{2} + 2t \right]_0^{\pi/2} = 3\sqrt{2} \cdot 2 \cdot \frac{\pi}{2} = \underline{3\sqrt{2}\pi} \approx 13,33$$

area:



$$A = 4 \int_{x=0}^{x=3} f(x) dx = 4 \int_{x=0}^{x=3} y dx$$

$$\text{dikt } \begin{cases} x = 3 \sin t \\ y = \frac{3}{2\sqrt{2}} \sin t \cos t \end{cases}$$

$$\frac{dx}{dt} = 3 \cos t$$

$$A = 4 \int_{t=0}^{t=\pi/2} \frac{3}{2\sqrt{2}} \sin t \cos t \cdot 3 \cos t dt = \frac{18}{\sqrt{2}} \int_0^{\pi/2} \sin t \cos^2 t dt$$

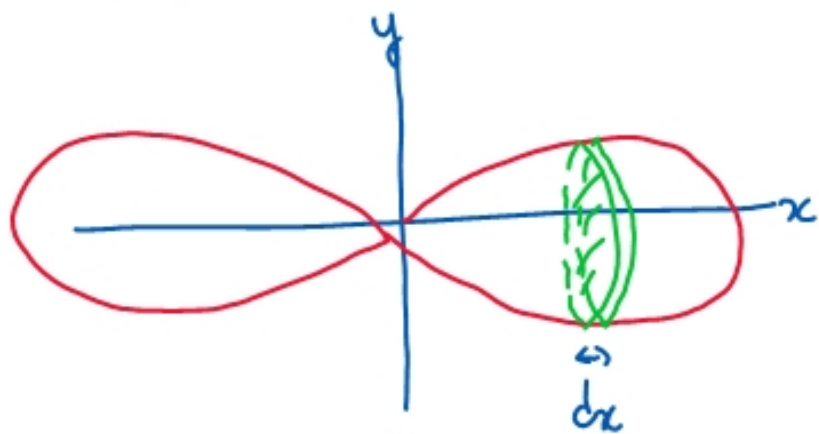
$$A = \frac{18}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \sin t \cos^2 t \, dt \quad \text{chyt } T = \cos t \quad \frac{dT}{dt} = -\sin t$$

$$\Rightarrow dt = -\frac{dT}{\sin t}$$

$$A = \left(\frac{18}{\sqrt{2}} \int_1^0 \cancel{\sin t} T^2 \left(-\frac{dT}{\cancel{\sin t}} \right) \right)$$

$$A = \frac{18\sqrt{2}}{2} \int_0^1 T^2 dT = 9\sqrt{2} \left[\frac{T^3}{3} \right]_0^1 = \frac{9\sqrt{2}}{3} = 3\sqrt{2} \approx \underline{4,2}$$

Volume de la surface obtenue par rotation autour de l'axe des x :



$$dV = \pi y^2 dx$$

$$V = 2 \int_{x=0}^3 \pi y^2 dx = 2\pi \int_{t=0}^{\frac{\pi}{2}} \frac{9 \sin^4 t \cos^2 t}{y^2} \underbrace{3 \cos t dt}_{dx}$$

$$V = \frac{27\pi}{4} \int_0^{\pi/2} \sin^2 t \cos^3 t dt \quad \text{let } T = \sin t \quad dT = \cos t dt$$

$$\left(V = \frac{27\pi}{4} \int_0^1 T^2 \cos^2 t \frac{dT}{\cos t} = \frac{27\pi}{4} \int_0^1 T^2 \cos t dT \quad \cos^2 t = 1 - \sin^2 t = 1 - T^2 \right)$$

$$V = \frac{27\pi}{4} \int_0^1 T^2 (1 - T^2) dT = \frac{27\pi}{4} \int_0^1 (T^2 - T^4) dT = \frac{27\pi}{4} \left[\frac{T^3}{3} - \frac{T^5}{5} \right]_0^1$$

$$V = \frac{27\pi}{4} \left[\frac{1}{3} - \frac{1}{5} \right] = \frac{27\pi}{4} \frac{5-3}{15} = \frac{27\pi}{9 \cdot 15} = \frac{9\pi}{2 \cdot 5} = \frac{9\pi}{10}$$

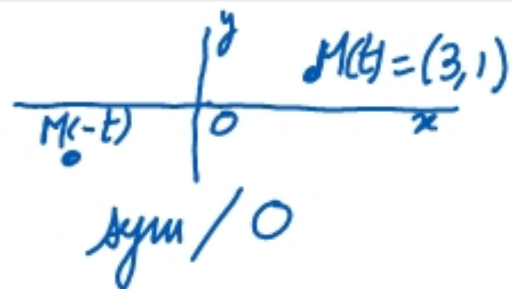
$$x = 3 \sin(t)$$

$$y = \frac{3}{2\sqrt{2}} \sin(t) \cos(t)$$

$$D_f = \mathbb{R} \quad \text{période } 2\pi$$

$$x(-t) = -x(t) \quad x \text{ est impaire}$$

$$y(-t) = -y(t) \quad y \text{ est impaire}$$



Étude sur $[0, \pi]$ + sym / 0

On a tous les pts pour $t \in [-\pi, \pi]$.

On récupère les pts pour $t \in [-\pi, 0]$

On a toute la courbe

• $x'(t) = 3 \cos(t)$ $x'(t) = 0 \Leftrightarrow t = \frac{\pi}{2} + k\pi$ enka 0 et π : en $\frac{\pi}{2}$

• $y'(t) = \frac{3}{2\sqrt{2}} (\cos^2 t - \sin^2 t) = \frac{3}{2\sqrt{2}} (1 - 2\sin^2 t) = \frac{3}{2\sqrt{2}} \cos(2t)$

$y'(t) = 0 \Leftrightarrow \cos(2t) = 0 \Leftrightarrow 2t = \frac{\pi}{2} + k\pi \Leftrightarrow t = \frac{\pi}{4} + \frac{k\pi}{2}$ $\frac{\pi}{4}$ et $\frac{3\pi}{4}$

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$x'(t)$	3	$+3\frac{\sqrt{2}}{2}$	0	$-3\frac{\sqrt{2}}{2}$	-3
x	0	$3\frac{\sqrt{2}}{2} \approx 2,1$	3	$3\frac{\sqrt{2}}{2}$	0
$y'(t)$	$3\frac{\sqrt{2}}{4}$	0	$-3\frac{\sqrt{2}}{4}$	0	$3\frac{\sqrt{2}}{4}$
y	0	$3\frac{\sqrt{2}}{8} \approx 0,5$	0	$3\frac{\sqrt{2}}{8}$	0
<p> $\text{pente} = \frac{y'}{x'} = \frac{\sqrt{2}}{4} \approx 0,35$ (I) $\frac{y'}{x'} = 0$ $\frac{y'}{x'} = \infty$ $\frac{y'}{x'} = 0$ $-\frac{\sqrt{2}}{4}$ \leftrightarrow \downarrow \leftrightarrow </p>					

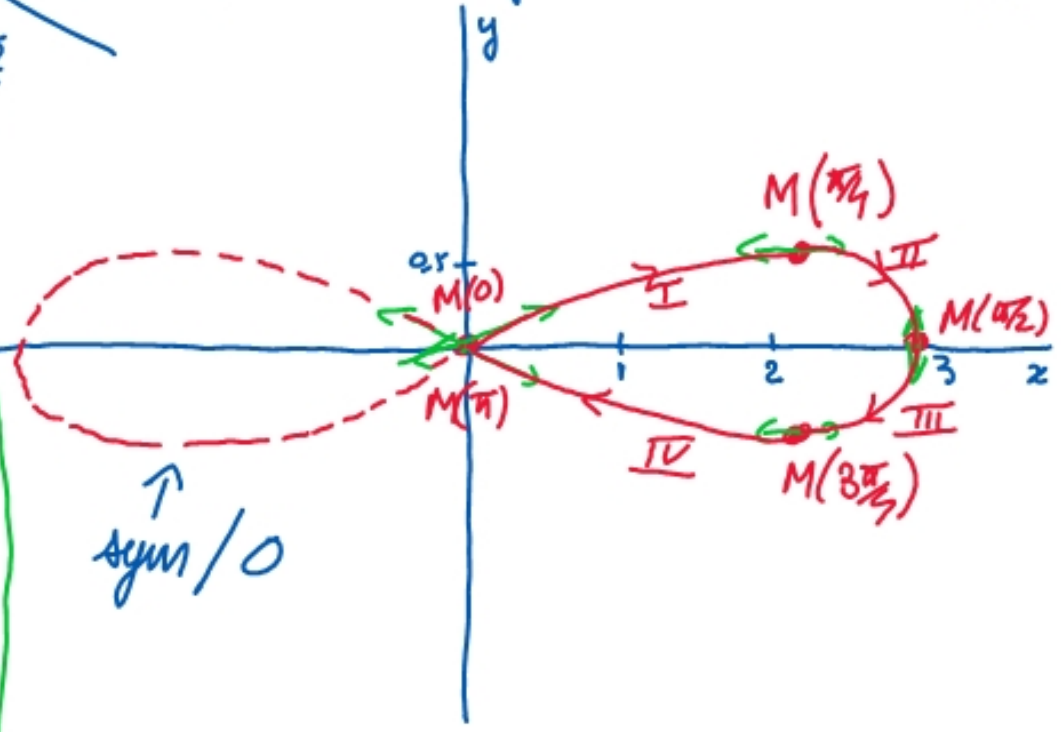
$$x(t) = 3 \sin t \quad x'(t) = 3 \cos t$$

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$$y\left(\frac{\pi}{4}\right) = \frac{3}{2\sqrt{2}} \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{8}$$

pente en un point: $\frac{y'}{x'}$
 (vektoren tangent $\vec{T} = (x', y')$)

longueur: $L = 4 \int_{t=0}^{t=\frac{\pi}{2}} \sqrt{x'(t)^2 + y'(t)^2} dt$
 $L = 4 \int_0^{\frac{\pi}{2}} \sqrt{9 \cos^2 t + \frac{9}{8} (2 \cos^2 t - 1)^2} dt$

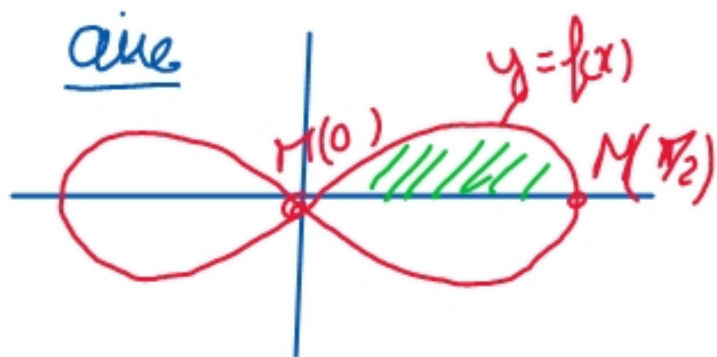


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$$L = 3\sqrt{2} \int_0^{\pi/2} (\cos(2t) + 1 + 1) dt = 3\sqrt{2} \int_0^{\pi/2} (\cos(2t) + 2) dt$$

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chgt $x = 3 \sin t$

$$y = \frac{3}{2\sqrt{2}} \sin t \cos t$$

$$\frac{dx}{dt} = 3 \cos t$$

$$A = 4 \int_{t=0}^{t=\pi/2} \underbrace{\frac{3}{2\sqrt{2}} \sin t \cos t}_{y(t)} \underbrace{3 \cos t}_{dx} dt = \frac{18}{\sqrt{2}} \int_0^{\pi/2} \sin t \cos^2 t dt$$

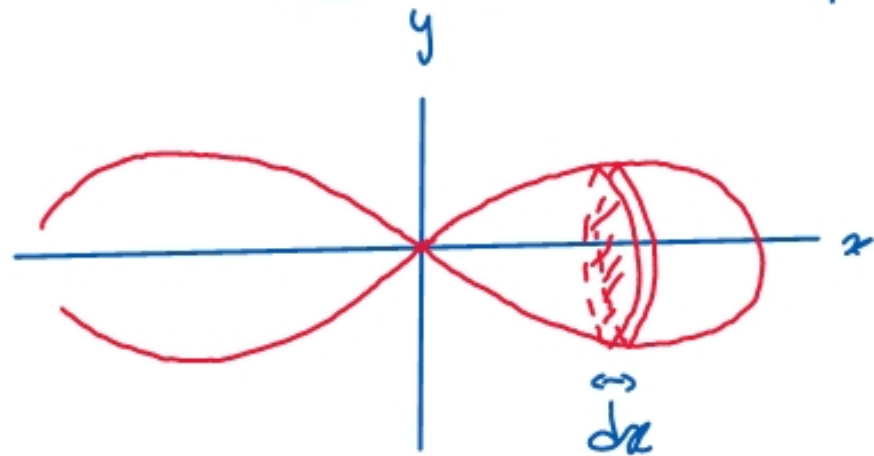
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$$A = \left(\frac{18}{\sqrt{2}} \int_{T=1}^{T=0} \sin t \, T^2 \left(-\frac{dt}{\sin t} \right) \right)$$

$$A = \frac{18\sqrt{2}}{2} \int_0^1 T^2 \, dT = 9\sqrt{2} \left[\frac{T^3}{3} \right]_0^1 = \frac{9\sqrt{2}}{3} = 3\sqrt{2} \approx 4,2$$

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$$V = \frac{27\pi}{4} \int_0^1 T^2 (1 - T^2) dT = \frac{27\pi}{4} \int_0^1 (T^2 - T^4) dT = \frac{27\pi}{4} \left[\frac{T^3}{3} - \frac{T^5}{5} \right]_0^1$$

$$V = \frac{27\pi}{4} \left[\frac{1}{3} - \frac{1}{5} \right] = \frac{27\pi}{4} \cdot \frac{2}{15} \quad \frac{27\pi}{2 \cdot 15} = \frac{9\pi}{2 \cdot 5} = \frac{9\pi}{10}$$