

Abstract

The MiMeS project demonstrated that a small fraction of massive stars (around 7%) presents large-scale, stable, generally dipolar magnetic fields at their surface. These fields that do not present any evident correlations with stellar mass or rotation are supposed to be fossil remnants of the initial phases of stellar evolution. These fossil fields result from the relaxation to MHD equilibrium states of initial fields resulting from a previous convective phase. In this poster, we present new theoretical results concerning the possible equilibrium field configurations where we generalize previous works by taking into account the rotation of stars. The properties of the fields are compared to those obtained when rotation is ignored. Consequences for magnetic fields in rotating intermediate-mass and massive stars and their stability are finally discussed.

Modelling and assumptions

- We study the axisymmetric case. The magnetic field \mathbf{B} and the velocity field \mathbf{v} can be written as

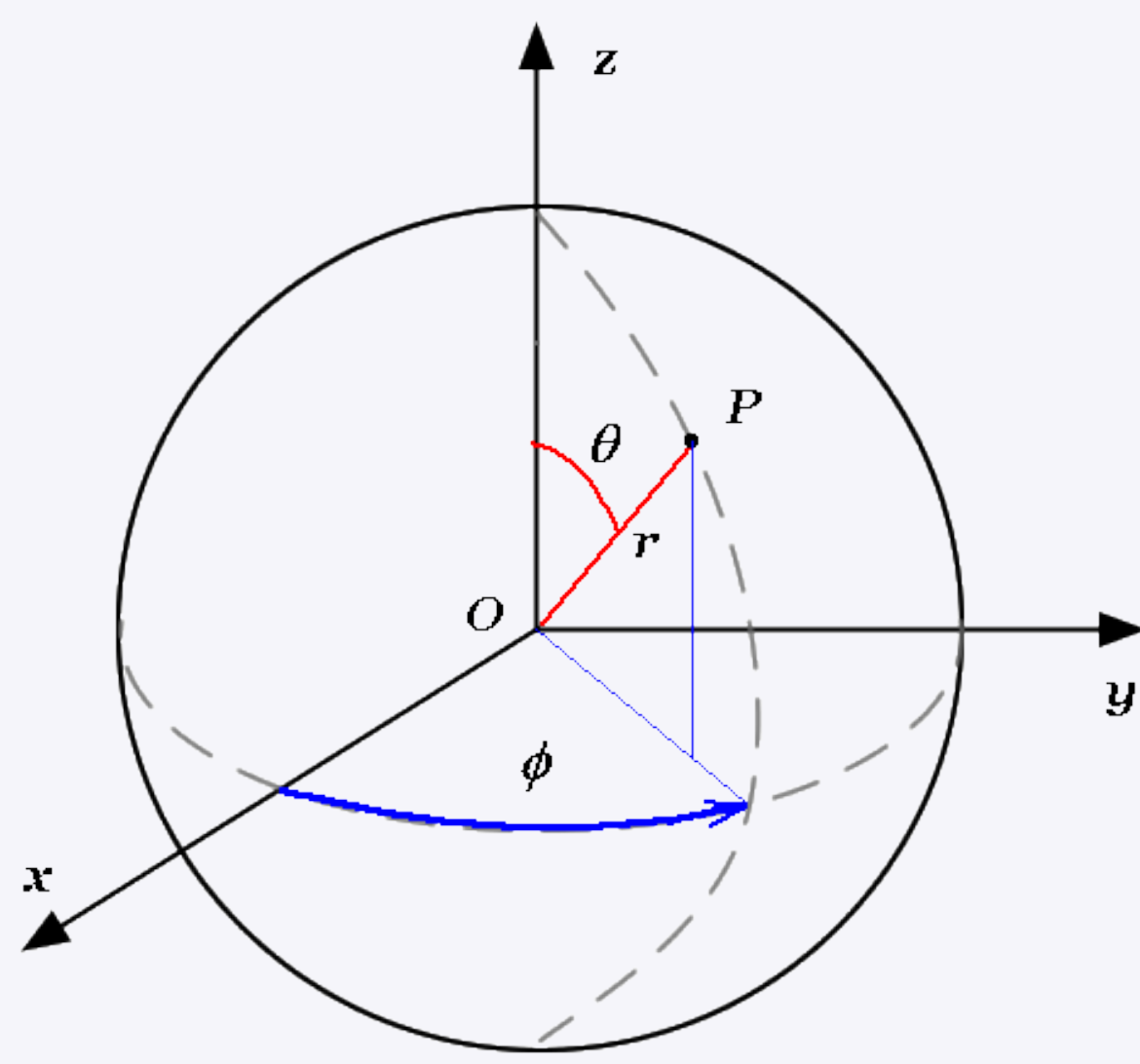
$$\mathbf{B} = \frac{1}{r\sin\theta} \nabla\Psi(r, \theta) \times \hat{e}_\phi + \frac{1}{r\sin\theta} F(r, \theta) \hat{e}_\phi$$

and

$$\mathbf{v} = \frac{1}{r\sin\theta} \nabla\Pi(r, \theta) \times \hat{e}_\phi + \frac{1}{r\sin\theta} E(r, \theta) \hat{e}_\phi,$$

where Ψ and Π are the poloidal flux functions and F and E are the toroidal potentials.

- To simplify the problem in a first step, ρ is assumed to be a constant (Prendergast 1956).
- We work with the usual spherical coordinate system (r, θ, ϕ) and $\{\hat{e}_k\}_{k=r,\theta,\phi}$ their usual vector-basis.



Summary

We have studied the properties of the fossil magnetic field in radiative zones considering the rotation of the star.

- The mode $l = 1$ is the mode of lowest energy.
- Since the equation that gives us Ψ is the same in both magneto-hydrostatic and magneto-hydrodynamic cases, the geometry of \mathbf{B} does not depend of the rotation rate of the star.
- The magnetic field \mathbf{B} and the velocity field \mathbf{v} in the radiative zone are fully determined by the properties of the convective progenitor and the boundary conditions.

References

- Reisenegger, A., Agkun, T., Mastrano, A., Marchant, P. 2013, *MNRAS*, 433, 2445
- Braithwaite, J. 2008, *MNRAS*, 386, 1947
- Duez, V., Mathis, S. 2010, *A&A*, 517, 10
- Prendergast, K. H. 1956, *ApJ*, 123, 498
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State of the art

- Poloidal and toroidal fields are unstable in stellar radiative zones (e.g. Tayler 1973 ; Markey and Tayler 1973).
- The best candidates for stable geometries are mixed poloidal-toroidal fields as it was shown numerically by Braithwaite (Braithwaite 2008).
- These mixed fields are stable when $\frac{E_{\text{mag,P}}}{E_{\text{mag,T}}} > \frac{E_{\text{mag,T}}}{E_{\text{grav}}}$ where $a = \frac{1.8}{\Gamma/\gamma-1}$ (Reisenegger 2013).
- The static axisymmetric case was studied by Duez and Mathis. The poloidal flux of \mathbf{B} is given by

$$\Delta^* \Psi + \frac{\lambda_1^2}{R^2} \Psi = -\mu_0 \rho r^2 \sin\theta \beta_0,$$

where $\Delta^* \Psi \equiv \partial_{rr} \Psi + \frac{\sin\theta}{r^2} \partial_\theta \left(\frac{1}{\sin\theta} \partial_\theta \Psi \right)$ and the values of the real coefficients λ_1 et β_0 are controlled by the initial magnetic helicity, $\mathcal{H} = \int_{\mathcal{V}} \mathbf{B} \cdot \mathbf{A} d\mathcal{V}$ (\mathbf{A} is the potential vector and \mathcal{V} is the volume of the star), and the mass conservation in each axisymmetric tube of flux, due to the stable stratification. The obtained lowest energy equilibrium is a dipolar stable non force-free configuration.

Relaxation of the magnetic fields in rotating star

- We study the relaxation of the magnetic field \mathbf{B} during the birth of a radiative zone in a purely convective star.
- We are looking for stable equilibrium states thanks to the variational problem

$$\delta E - a_{11} M_\Psi - a_{21} \mathcal{H} - a_{31} \mathcal{H}_v - a_{40} \mathcal{L} = 0,$$

where

$$- M_\Psi = \int_{\mathcal{V}} \rho \Psi d\mathcal{V},$$

$$- \mathcal{H} = \int_{\mathcal{V}} \mathbf{B} \cdot \mathbf{A} d\mathcal{V},$$

$$- \mathcal{H}_v = \int_{\mathcal{V}} \mathbf{B} \cdot \mathbf{v} d\mathcal{V},$$

$$- \mathcal{L} = \int_{\mathcal{V}} r \sin\theta \rho v_{\text{phi}} d\mathcal{V}.$$

- Equation* - The variational problem gives us the equation that governs Ψ

$$\Delta \Psi + a \Psi = b.$$

- First results* - This equation has the same form than the equation obtained by Duez and Mathis (2010). Hence, we deduce that
 - The mode $l = 1$ is still the mode of lowest energy.
 - The geometry of the magnetic field at the surface will be the same, i.e. it does not depend on the rotation rate. This result is coherent with the observations.

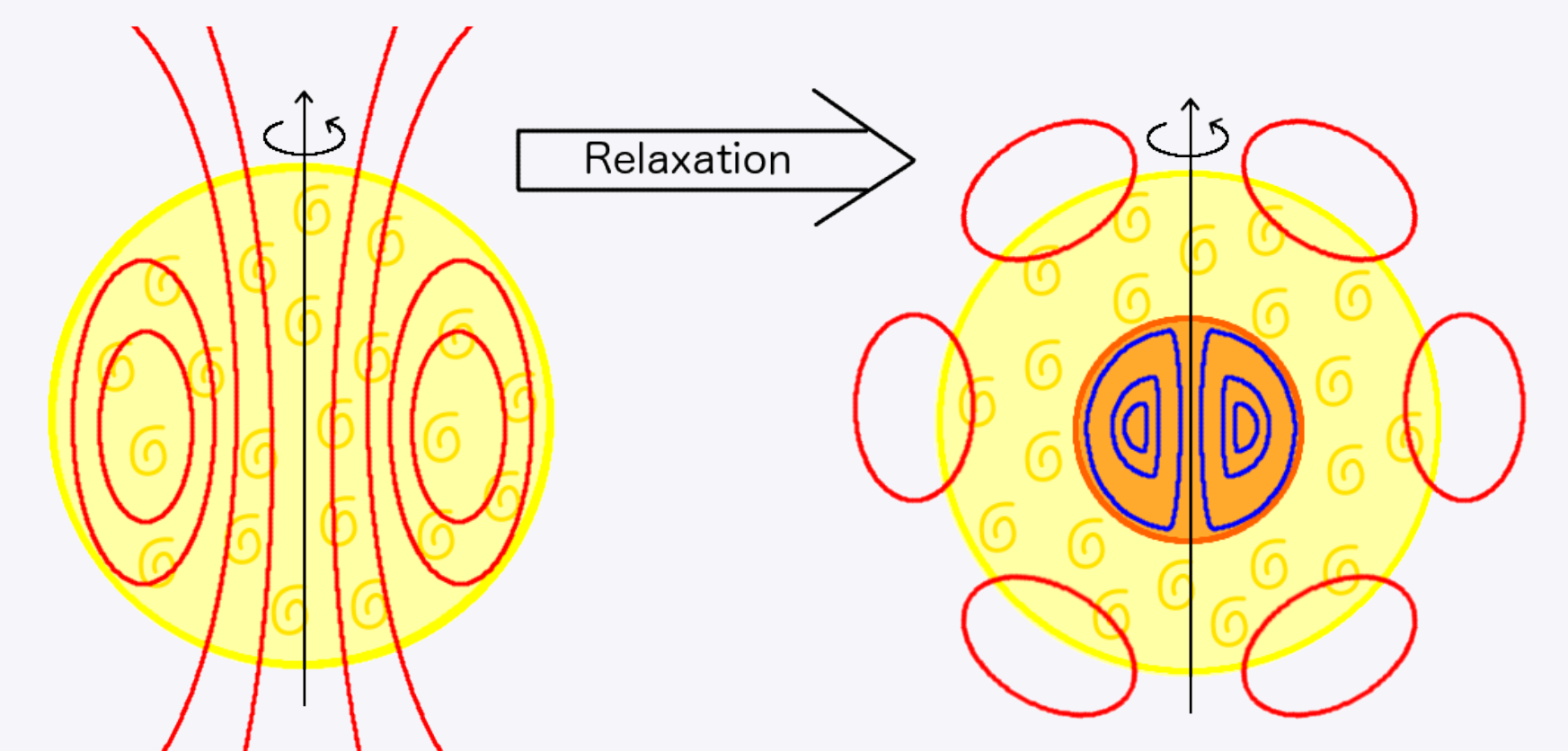


Figure 1: Relaxation of the magnetic field during the apparition of the internal radiative zone in a purely convective star.

Properties of resulting fossil field

- For the first time, the fossil magnetic field can be fully determined thanks to the properties of the convective progenitor and the boundary conditions.
- We choose the case of initially confined configurations $\mathbf{B} \cdot \hat{e}_r = 0$ and $\mathbf{B} \cdot \hat{e}_\theta = 0$.
- The whole determination of \mathbf{B} and \mathbf{v} allows to predict the complete properties of resulting equilibrium

$$- E_{\text{mag}} = -\frac{1}{2\mu_0(1-a_{31}^2)} \left[a_{21} \mathcal{H}^c + \left(a_{11} - \frac{a_{21} a_{31} a_{40}}{\rho} \right) M_\Psi - \frac{a_{31}^2 a_{40}}{a_{20} \rho} (1 - a_{31}^2) \mathcal{L} \right],$$

$$- \frac{E_{\text{mag,P}}}{E_{\text{mag,T}}} = \frac{a_{21} \mathcal{H}^c + 2a_{11} M_\Psi}{a_{21} \mathcal{H}^c - 2 \frac{a_{31}^2 a_{40}}{\rho} \mathcal{L} + 2 \frac{a_{21} a_{31} a_{40}}{\rho} M_\Psi},$$

$$- E_{\text{cin}} = \frac{a_{31} \rho}{2} \left[\mathcal{H}_v^c + \frac{a_{40}}{a_{31} \rho} \mathcal{L} \right].$$