

Abstract

Modelling and assumptions

- We study the axisymmetric case. The magnetic field \mathbf{B} and the velocity field \mathbf{v} can be written as

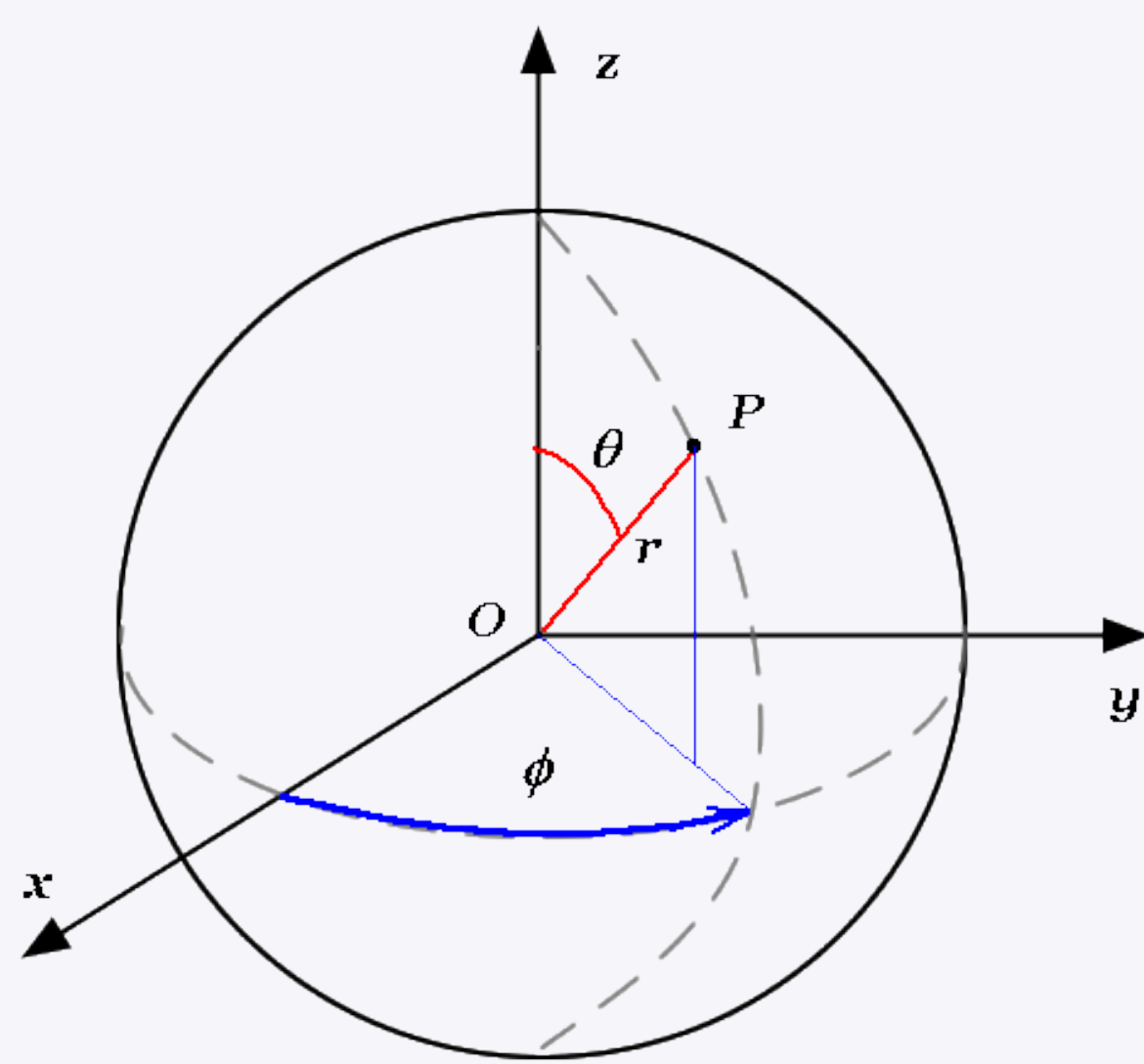
$$\mathbf{B} = \frac{1}{r \sin \theta} \nabla \Psi(r, \theta) \hat{e}_\phi \times \frac{1}{r \sin \theta} F(r, \theta) \hat{e}_\phi$$

and

$$\mathbf{v} = \frac{1}{r \sin \theta} \nabla \Pi(r, \theta) \hat{e}_\phi \times \frac{1}{r \sin \theta} E(r, \theta) \hat{e}_\phi$$

where Ψ and Π are the poloidal fluxes and F and E are the toroidal potentials.

- Unlike the article written by Duez and Mathis, ρ is constant (Pendergast 1956).
- We work with the usual spherical coordinate system (r, θ, ϕ) and $\{\hat{e}_k\}_{k=r,\theta,\phi}$.



Summary

We have studied the properties of the fossil magnetic field in radiative zones considering the rotation of the star.

- The mode $l = 1$ is the mode of the lowest energy.
- Since the equation that gives us Ψ is the same in both magneto-hydrostatic and magneto-hydrodynamic cases, the geometry of \mathbf{B} does not depend on the rotation rate of the star.
- The magnetic field \mathbf{B} and the velocity field \mathbf{v} in the radiative zone are fully determined by the properties of the convective progenitor and the boundary conditions.

References

- [1] Reisenegger, A., Agkun, T., Mastrano, A., Marchant P. 2013, *MNRAS*, 433, 2445
- [2] Braithwaite, J. 2008, *MNRAS*, 386, 1947-1958
- [3] Duez, V., Mathis, S. 2010, *A&A*, 517
- [4] Pendergast, K. H. 1956, *ApJ*, 123, 498
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State of the art

- Poloidal and toroidal fields are unstable in the radiative zone (e.g. Tayler 1973; Markey and Tayler 1973).
- The best candidates for stable geometries are mixed poloidal-toroidal fields as it was shown numerically by Braithwaite.
- These mixed fields are stable when $\frac{E_{mag,P}}{E_{mag,T}} > 2a \frac{E_{mag,T}}{E_{grav}}$ (Reisenegger 2013).
- The axisymmetric case was studied in magneto-hydrostatic by Duez and Mathis. The poloidal flux of \mathbf{B} is given by

$$\Delta^* \Psi + \frac{\lambda_1^2}{R^2} \Psi = -\mu_0 \rho r^2 \sin^2 \theta \beta_0$$

where $\Delta^* \Psi \equiv \partial_{rr} \Psi + \frac{\sin \theta}{r^2} \partial_\theta \left(\frac{1}{\sin \theta} \partial_\theta \Psi \right)$ and the values of the real coefficients λ_1 and β_0 are controlled by the helicity and the mass conservation in each axisymmetric tube of flux.

Relaxation of the magnetic field

- We study the relaxation of the magnetic field \mathbf{B} during the apparition of a radiative zone in a purely convective star.
- We are looking for stable equilibrium states thanks to the variational problem

$$\delta E - a_{11} I_{11} - a_{21} I_{21} - a_{31} I_{31} - a_{40} I_{40} = 0.$$

where I_{11} , I_{21} , I_{31} and I_{40} represent the conservation of the mass in each flux tube, the helicity, the cross-helicity and the angular momentum.

- Equation* - The variational problem gives us the equation that governs Ψ

$$\Delta^* \Psi + a \Psi = b$$

- First results* - This equation has the same form than the equation of the article written by Duez and Mathis. Hence we can deduce that

- The mode $l = 1$ is still the mode of lowest energy.
- The geometry of the magnetic field at the surface will be the same, i.e. it does not depend on the rotation rate. This result is consistent with the observations.

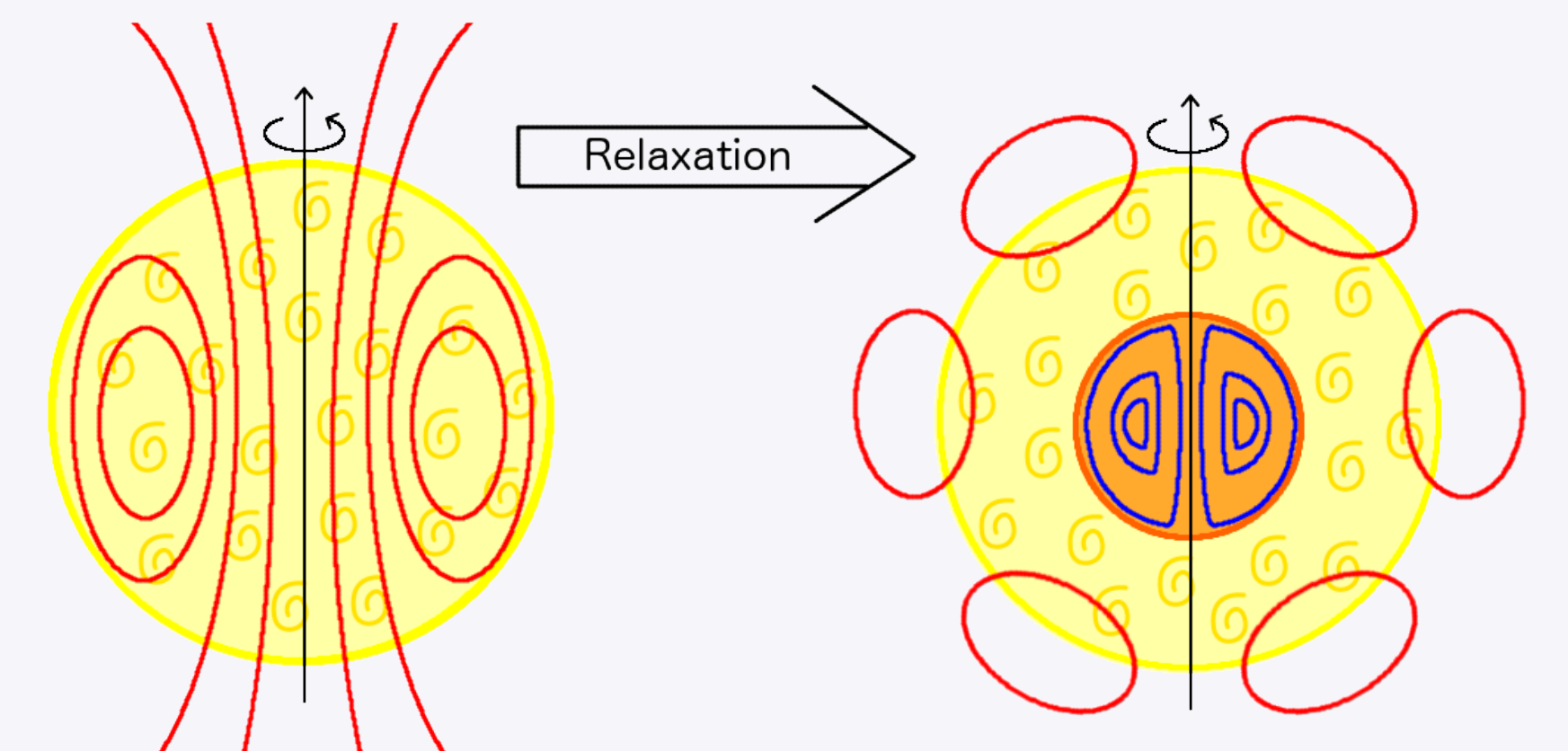


Figure 1: Relaxation of the magnetic field during the apparition of the internal radiative zone in a purely convective star.

Ratio $E_{mag,P} / E_{mag,T}$

- The fossil magnetic field can be fully determined thanks to the properties of the convective progenitor and the boundary conditions.

- These conditions are

$$\mathbf{B} \cdot \hat{e}_r = 0 \text{ and } \mathbf{B} \cdot \hat{e}_\theta = 0$$

- The whole determination of \mathbf{B} and \mathbf{v} gives us the ratio between the toroidal and the poloidal components of E_{mag}

$$\frac{E_{mag,P}}{E_{mag,T}} = \frac{a_{21} \mathcal{H}^c + 2a_{11}}{a_{21} \mathcal{H}^c - 2 \frac{a_{31}^2 a_{40}}{\rho} I_{40} + 2 \frac{a_{21} a_{31} a_{40}}{\rho} I_{11}}$$