

TITLE

Titre



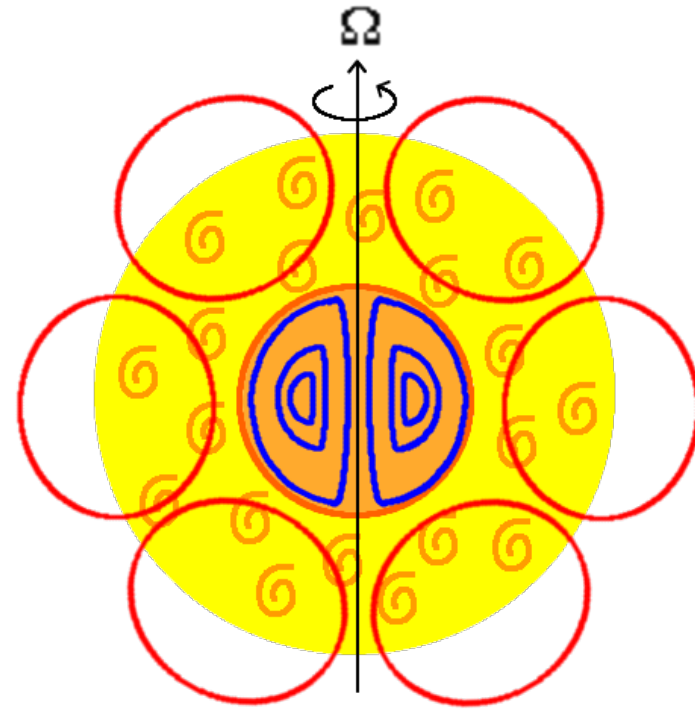
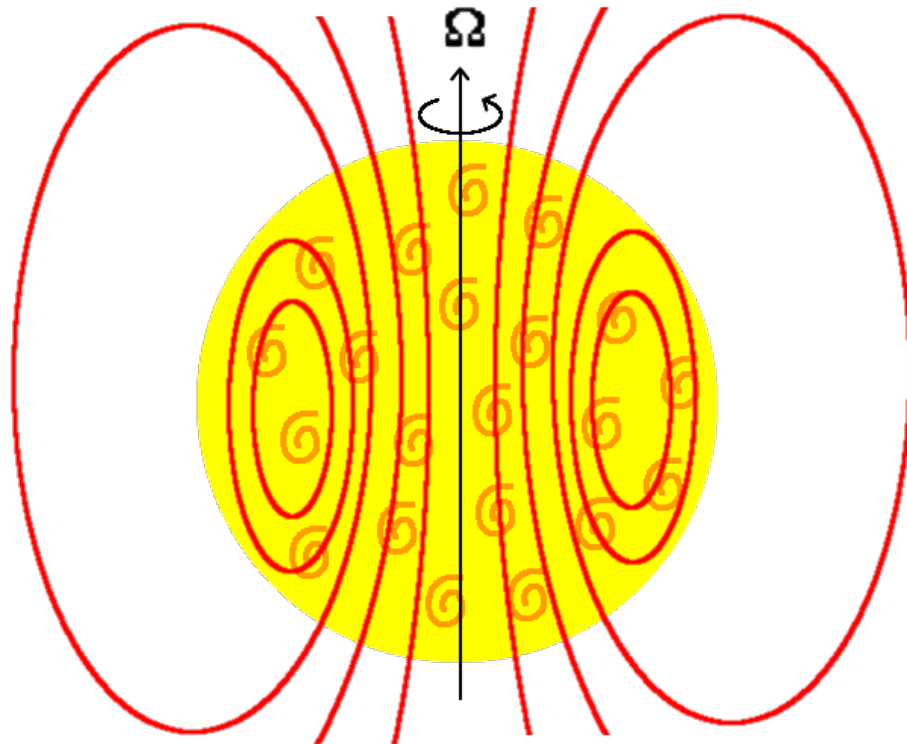
# Magnetic fields in stellar radiative zones





# Relaxation

Relaxation →



 Convection zone

 Dynamo field

 Radiation zone

 Fossil field

# Relaxation

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- *Systeme*

$$\left\{ \begin{array}{l} \Delta^*\Psi + \alpha^2\Psi = \beta - \gamma \\ \Pi = a_{31}\Psi \\ F = -\alpha\Psi - \frac{\gamma}{\alpha}r^2\sin^2\theta \\ G = -\alpha a_{31}\Psi - \frac{\gamma}{a_{31}\alpha}r^2\sin^2\theta \end{array} \right.$$

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$$\Delta^*\Psi \equiv \partial_{rr}\Psi + \frac{\sin\theta}{r^2}\partial_\theta\left(\frac{1}{\sin\theta}\partial_\theta\Psi\right)$$

# Solution

$$\begin{aligned} \Psi(r, \theta) = & \sin^2 \theta \left[ \sum_{l=0}^{\infty} K_1^l \alpha r j_{l+1}(\alpha r) C_l^{3/2}(\alpha r) \right. \\ & + \alpha r (\beta - \gamma) y_1(\alpha r) \int_{R_i}^r j_1(\alpha \xi) \xi^3 d\xi \\ & \left. + \alpha r (\beta - \gamma) j_1(\alpha r) \int_r^{R_s} y_1(\alpha \xi) \xi^3 d\xi \right] \end{aligned}$$

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$\alpha, \beta, \gamma, K_1^l$  ?



# Conditions

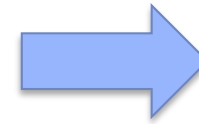
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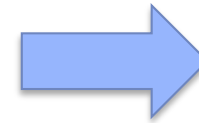


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Initial conditions :

$$\mathcal{H}^c = -\frac{1}{4\alpha\pi\rho} \int_{\mathcal{V}} B_T^2 + B_P^2 d\mathcal{V} + \left[ \frac{\gamma}{\alpha} (1 - a_{31}^2)^2 - \frac{\beta}{\alpha} \right] M_\Psi - \frac{\gamma}{\alpha^2} a_{31} \mathcal{L}$$

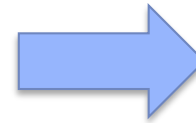
$$\mathcal{H}_v^c = \frac{a_{31}}{4\pi\rho} \int_{\mathcal{V}} B_T^2 + B_P^2 d\mathcal{V} + \frac{\gamma}{a_{31}} (1 - a_{31}^2)^2 M_\Psi - \frac{\gamma}{\alpha} (1 - a_{31}^2) \mathcal{L}$$

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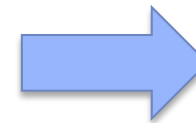
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$$\beta, \gamma, a_{31}$$

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# Full Solution

$$\Psi(r, \theta) = \sin^2 \theta (\beta - \gamma) \frac{r}{\alpha^3} [f_\alpha(R_s) j_1(\alpha r) - f_\alpha(R_i) y_1(\alpha r) + \alpha r]$$

$$f_\alpha(r) = -(\alpha^3 r^3 - 3) \sin(\alpha r) - 3\alpha r \cos(\alpha r)$$

# Links between helicities and energies

$$E_{\text{mag}} = -4\pi\rho\frac{\alpha}{2\mu_0} \left( \mathcal{H}^c + \left[ \frac{\beta}{\alpha} - \frac{\gamma}{\alpha} (1 - a_{31}^2) \right] M_{\Psi} + \frac{\gamma}{\alpha^2} a_{31} \mathcal{L} \right)$$

$$E_{\text{cin}} = \frac{a_{31}\rho}{2\sqrt{4\pi\rho}} \left[ \mathcal{H}_v^c - \frac{\gamma}{\alpha} (1 - a_{31}^2) \frac{\sqrt{4\pi\rho}}{a_{31}^2} \mathcal{L} \right]$$

# Velocity field profile

$$\begin{cases} \mathbf{v}_P &= \frac{a_{31}}{\sqrt{4\pi\rho}} \mathbf{B}_P \\ \mathbf{v}_T &= \frac{a_{31}}{\sqrt{4\pi\rho}} \mathbf{B}_T + \frac{\gamma}{\alpha} r \sin\theta \left( a_{31} - \frac{1}{a_{31}} \right) \hat{e}_\varphi \end{cases}$$

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**Solid rotation**

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**Aligned with magnetic field**

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# Conclusion

