

TD 9

1. Un atome dans l'espace libre

1)  $E_i = \hbar \omega_A$  ;  $E_f = 0 + \hbar c |\vec{k}| = \hbar c |\vec{k}|$   
 $\frac{1}{\hbar} \omega = c|\vec{k}|$

2)  $\rho(E) dE = \rho(k) d^3k = \frac{L^3}{(2\pi)^3} 4\pi k^2 dk = \frac{L^3}{(2\pi)^3} \frac{dE}{\hbar c}$   
 et  $k^2 = \left(\frac{E}{\hbar c}\right)^2$

$\Rightarrow \rho(E) = \frac{L^3}{2\pi^2} \cdot \frac{E^2}{(\hbar c)^3}$

On prend un spin nul

3)  $\Gamma = \frac{2\pi}{\hbar} \rho(E_f) | \langle E_f = E_f | \hat{V} | E_i \rangle |^2$

$= \frac{2\pi}{\hbar} \frac{L^3}{2\pi^2} \frac{(\hbar \omega_A)^2}{(\hbar c)^3} \cdot \left(\frac{\hbar \omega_A}{2\hbar c L^3}\right) d^2$

$\Gamma = \frac{\omega_A^2 d^2}{2\pi \epsilon_0 \hbar c^3}$

2. Un atome fermionique en présence d'une mer de Fermi

1)  $\Psi_f(\vec{r}) = \frac{1}{\sqrt{V}} e^{i\vec{q} \cdot \vec{r}}$

$E_q = \frac{\hbar^2 q^2}{2m}$

$\vec{q}$  quantité

$\begin{cases} q_x = \frac{2\pi n_x}{L} \\ q_y = \frac{2\pi n_y}{L} \\ q_z = \frac{2\pi n_z}{L} \end{cases}$

$(n_x, n_y, n_z) \in \mathbb{Z}^3$

1. Etude générale  $\forall d$

1)  $\sum_{N+1} V | \Delta \rangle = \sum_{N+1} | \Delta \rangle \times \sum_{\vec{q}_1, \vec{q}_2, \vec{q}_3} | \vec{q}_1, \vec{q}_2, \vec{q}_3 \rangle ; g_{\vec{q}_i} > 1k >$

2)  $\Delta \in S = \vec{q}_1, \dots, \vec{q}_N$  non deux à deux distincts

$\exists (i, j) \in \{1, \dots, N\}^2, \vec{q}_i = \vec{q}_j$

$| i \rangle = \sqrt{N+1} | A_{N+1} | i \rangle = \frac{1}{\sqrt{N+1}} \sum_{\vec{q} \in S} | \vec{q}_1, \dots, \vec{q}_N, \vec{q}_{N+1} \rangle ; \vec{q}_{N+1} > E(\vec{q})$

fermions  $\Rightarrow$  pas dans le  $\vec{q}$  état quantique  $\Rightarrow$  si  $\vec{q}_1, \dots, \vec{q}_N$  non 2 à 2 distincts

Principe de Pauli

tient compte de  $\delta(\vec{q}_i - \vec{q}_j) - \delta(\vec{q}_i + \vec{q}_j)$

Permutation  $\rightarrow$  néglige pas compte de  $\delta(\vec{q}_i + \vec{q}_j) \rightarrow \pm 1$

somme nulle

$$3) \langle \delta | \hat{V}_{N+1} | i \rangle = (N+1)! \langle \delta | \hat{A}_{N+1} \hat{V}_{N+1} \hat{A}_{N+1}^\dagger | i \rangle$$

$$= \frac{A_{N+1}}{A_{N+1}} = \hat{A}_{N+1} \hat{V}_{N+1}$$

$$= (N+1)! \langle \delta | \hat{A}_{N+1} \hat{V}_{N+1} | i \rangle$$

$$4) \langle \delta | \hat{V}_{N+1} | i \rangle = (N+1)! \langle \delta | \hat{A}_{N+1} \hat{V}_{N+1} | i \rangle = \frac{(N+1)!}{(N+1)!} \langle \delta | \hat{A}_{N+1} \sum_k (g_k \hat{a}_k^\dagger + g_k \hat{a}_k) | i \rangle$$

$$= \sum_k \langle \delta | \hat{A}_{N+1} \hat{V}_{N+1} | i \rangle = \sum_k \langle \delta | \hat{A}_{N+1} \hat{V}_{N+1} | i \rangle$$

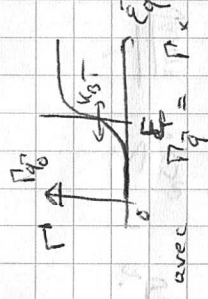
$$5) (i) \Gamma_{q_0}^{N+1} = \frac{2\pi}{\hbar} \rho(E_q = \epsilon_1) | \langle \delta | \hat{V}_{N+1} | i \rangle |^2$$

$$= \frac{2\pi}{\hbar} \frac{L_A}{2\pi^2} \left( \frac{\hbar^2 q_0}{2m} \right)^2 \frac{1}{(\hbar c)^3} \frac{V_0^2 d^2}{L_0^3} = \left( \frac{\hbar^2 q_0^2}{2m^2 c^3} \right)$$

=  $\Gamma$  (de la 1<sup>ère</sup> partie)

(ii)  $\Gamma_{q_0}^{N+1} = \alpha$  car  $\langle \delta | \hat{V}_{N+1} | i \rangle = 0$  si  $q_0$  est égal à à l'un des  $q_1, \dots, q_N$

$$6) \Gamma_{q_0}^{N+1} = \sum_{q_0} \epsilon_{q_0}^3 \times \frac{d^2}{\hbar^4 \pi^2 \epsilon c^3}$$



$$7) \Gamma' = \sum_q \Gamma_q P_q$$

$$8) \Gamma' = \sum_q \Gamma_q P_q \text{ avec } \Gamma_q = \Gamma \times \text{probabilité} + 0 \times \text{probabilité} = \Gamma \times (1 - n_q) + 0 n_q$$

## 2. Application à l'équilibre thermodynamique

$$1) P_q = \frac{1}{\sum_q e^{-\beta \epsilon_q}} e^{-\beta \epsilon_q} = \frac{e^{-\beta \epsilon_q}}{\mathcal{Z}}$$

$$2) \Gamma' = \Gamma \frac{\sum_q e^{-\beta \epsilon_q} (1 - n_q)}{\mathcal{Z}} = \frac{\Gamma}{\mathcal{Z}} \sum_q e^{-\beta \epsilon_q} (1 - n_q) = \frac{\Gamma}{\mathcal{Z}} \sum_q e^{-\beta \epsilon_q} - \frac{\Gamma}{\mathcal{Z}} \sum_q e^{-\beta \epsilon_q} n_q$$

$$3) \mathcal{Z} = \sum_n e^{-\beta \epsilon_n} = \int d^3 p d^3 q \frac{1}{h^3} e^{-\beta \frac{\hbar^2 p^2}{2m}} = \frac{L^3}{(2\pi\hbar)^3} \times \left( \frac{\sqrt{2\pi m k_B T}}{\hbar} \right)^3 = \frac{L^3}{\lambda_{DB}^3}$$

avec  $\lambda_{DB} = \left( \frac{2\pi\hbar^2}{m k_B T} \right)^{1/2}$

$$4) \frac{P'}{P} = \frac{\lambda_{DB}^3}{V} e^{-\beta \mu} N_g = \rho_g \lambda_{DB}^3 e^{-\beta \mu}$$

$$5) T \gg T_F, N = \sum_i e^{-\beta(\epsilon_i - \mu)} = e^{\beta \mu} Z = e^{\beta \mu} \frac{V}{\lambda_{DB}^3}$$

$$\boxed{\rho_g \lambda_{DB}^3 e^{-\beta \mu} = 1}$$

$$6) T \ll T_F \quad E_F = \frac{\hbar^2}{2m} (3\pi^2 \rho)^{2/3} = k_B T_F$$

$$\left\{ \begin{array}{l} \rho_g = \left( \frac{2m(E_F)}{\hbar^2} \right)^{3/2} \frac{1}{3\pi^2} \\ \lambda_{DB} = \left( \frac{2\pi \hbar^2 \beta}{m k_B T} \right)^{1/2} \\ e^{-\beta \mu} = e^{-\beta E_F} \end{array} \right.$$

$$\Rightarrow \frac{P'}{P} = \frac{1}{3\pi^2} \left( \frac{4\pi T_F}{T} \right)^{3/2} e^{-\beta E_F}$$