

1)  $\Gamma = \frac{2\pi}{\hbar} \underbrace{|\langle E_0 | V | \Phi \rangle|^2}_{= v^2} \underbrace{\rho(E_k = E_0)}_{= \frac{1}{S}} \quad \text{car } \rho(E) dE = \frac{dE}{S}$

$\Gamma = \frac{2\pi}{\hbar} v^2$

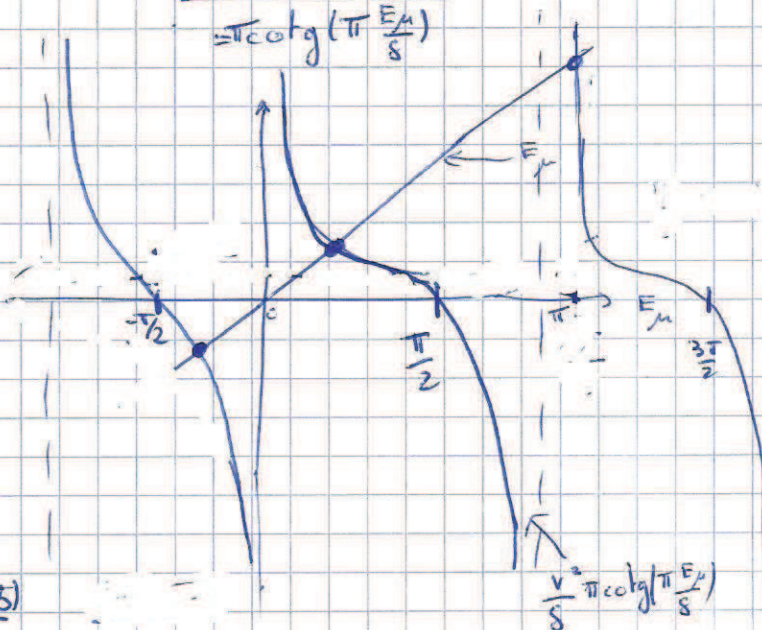
2)  $\rho E \ll 1$

3) •  $\langle k | H | \Psi_\mu \rangle = \langle k | H (\alpha | \Phi \rangle + \sum_k \beta_k | k \rangle) = \alpha \times (0 + v) + \dots k S \beta_k = \alpha v + k S \beta_k$   
 $= E_\mu \langle k | (\alpha | \Phi \rangle + \sum_k \beta_k | k \rangle) = E_\mu \beta_k$

•  $\langle \Phi | H | \Psi_\mu \rangle = \sum_k \beta_k v = E_\mu \alpha$

$\Rightarrow \sum_k \frac{\alpha v \beta_k}{E_\mu - k S} = E_\mu \alpha \quad \Rightarrow \left[ \sum_k \frac{v^2}{E_\mu - k S} = E_\mu \right]$

4)  $\Rightarrow \frac{v^2}{S} \sum_k \frac{1}{\left( \frac{E_\mu}{S} - k \right)} = E_\mu$   
 $= \pi \cot\left(\pi \frac{E_\mu}{S}\right)$



5) Sans perturbation  $E_\mu = k S \Rightarrow \cot\left(\pi \frac{E_\mu}{S}\right) = \infty \gg 1$   
 $\Rightarrow \frac{E_\mu S}{\pi v^2} \gg 1 \Rightarrow E_\mu \gg \hbar \Gamma$

$\Delta E \sim \hbar \Gamma$

6)  $|\langle \Phi | \Psi_\mu \rangle| = |\alpha| \quad \text{et} \quad \sum_k |\beta_k|^2 + |\alpha|^2 = 1$

$\left(\frac{\cos}{\sin}\right)' = \frac{-\sin^2 - \cos^2}{\sin^2} = -\frac{1}{\sin^2}$

$\Rightarrow |\alpha|^2 = 1 - \sum_k \frac{|\alpha v|^2}{\left(\frac{E_\mu}{S} - k\right)^2}$   
 $\Rightarrow |\alpha|^2 = \frac{1}{\left(1 + \sum_k \left(\frac{v}{\frac{E_\mu}{S} - k}\right)^2\right)^{1/2}}$

De  $\hat{m} \quad |\beta_k| = \frac{v}{\left(\frac{E_\mu}{S} - k\right)} \left[1 + \sum_k \left(\frac{v}{\frac{E_\mu}{S} - k}\right)^2\right]^{-1/2}$



$$7. \quad \langle \Phi | \Psi_\mu \rangle = \frac{1}{\left[ 1 + \sum_k \left( \frac{v/\delta}{E_\mu/\delta - k'} \right)^2 \right]^{1/2}}$$

En dérivant par rap à z,  $\sum_k - (z - k)^{-2} = -\pi^2 (1 + \cotan^2(\pi z))$

$$\hookrightarrow \langle \Phi | \Psi_\mu \rangle = \frac{1}{\left[ 1 + \frac{v^2}{\delta^2} \left( \pi^2 + \pi^2 \left( \frac{E_\mu \delta}{\pi v^2} \right)^2 \right) \right]^{1/2}} \quad \text{car} \quad E_\mu = \frac{v^2}{\delta} \pi \cotan(\pi z)$$

$$= \frac{1}{\left( 1 + \frac{\pi^2 v^2}{\delta^2} + \frac{E_\mu^2}{v^2} \right)^{1/2}}$$

$\approx \frac{\hbar^2 \pi^2}{4v^2}$

$$8. \quad |\Phi\rangle = \sum_\mu \langle \Phi | \Psi_\mu \rangle |\Psi_\mu\rangle e^{i\Phi_\mu}$$

$$|\Psi(t)\rangle = \sum_\mu e^{i\Phi_\mu} \langle \Psi_\mu | \Phi \rangle |e^{-i\frac{E_\mu t}{\hbar}} |\Psi_\mu\rangle$$

$$P = |\langle \Phi | \Psi(t) \rangle|^2 = \left| \sum_\mu \langle \Psi_\mu | \Phi \rangle^2 e^{-i\frac{E_\mu t}{\hbar}} \right|^2 = \left| \sum_\mu \frac{e^{-i\frac{E_\mu t}{\hbar}}}{1 + \frac{\hbar^2 \pi^2}{4v^2} + \frac{E_\mu^2}{v^2}} \right|^2$$

$$\frac{\hbar^2 \pi^2}{4v^2} = \frac{v^2}{4\delta^2} = \frac{v^2}{4\delta} \xrightarrow{\delta \rightarrow 0} 1$$

$$P = \left| \sum_\mu \frac{e^{-i\frac{E_\mu t}{\hbar}}}{\frac{\hbar^2 \pi^2}{4v^2} + \frac{E_\mu^2}{v^2}} \right|^2$$

$$\int \frac{e^{iu\tau/\hbar}}{u^2 + 1/\hbar} du = 2i\pi \sum_{\substack{k \text{ pôles} \\ \text{Im}(k) > 0}} \text{Res} \left( \frac{e^{iu\tau/\hbar}}{u^2 + 1/\hbar}, \text{pôle } k \right)$$

$$\text{car Res} = \lim_{u \rightarrow \text{pôle}} (u - \text{pôle}) \times \frac{e^{iu\tau/\hbar}}{u^2 + 1/\hbar}$$

$$P = \left| \int \frac{1}{\delta} \frac{e^{-iE\tau/\hbar}}{\frac{\hbar^2 \pi^2}{4v^2} + \frac{E^2}{v^2}} dE \right|^2 = \left( \frac{1}{\delta \frac{\hbar^2 \pi^2}{v^2}} \right)^2 \left| \int \frac{e^{-iE\tau/\hbar}}{1/\hbar + \left( \frac{E^2}{\hbar^2 \pi^2} \right)} dE \right|^2$$

$$P_\mu = \frac{\hbar}{\delta} \left( \frac{\hbar}{\delta \frac{\hbar^2 \pi^2}{v^2}} \right)^2 \left| \int \frac{e^{\frac{1}{\hbar} \tau u}}{1/\hbar + u^2} du \right|^2$$

intégrale de  $-\infty$  à  $+\infty$  donc ok

9 pour  $u = i/2$   
(th des résidus)

$$= \left( \frac{v^2}{\delta \hbar \pi} \right)^2 \left| 2i\pi \frac{e^{-\tau/\hbar}}{i} \right|^2$$

$$P = \frac{2\pi}{\delta \frac{\hbar^2 \pi^2}{v^2}} e^{-\tau/\hbar}$$

$$\bar{g} = \left| \sum_{\mu} \frac{e^{-i E_{\mu} t}}{E_{\mu}^2 + \frac{\hbar^2 \mu^2}{2m}} \right|$$

$$E_{\mu} = \sqrt{\frac{\hbar^2 \mu^2}{2m}}$$

Résonance si tous les termes de la somme sont en phase

$$\Leftrightarrow \mu \delta t = 2\pi \frac{\hbar}{h}$$

$$\Rightarrow \boxed{N \approx 30} \quad \text{si } \mu = 1, \delta t = 2\pi \hbar = 0,2 \hbar N t$$