Two-photon interference

Due 25 march 2014

The Hong Ou Mandel (HOM) experiment, Phys. Rev. Lett. **59**, 2044 (1987) (original paper attached), is about what happens when identical, single photons arrive at the 2 inputs of a beamsplitter. "Identical" means having the same frequency, spatial mode and polarization. We label a and b the input ports and e and f the output ports. The transmission and reflection coefficients of the electric field are $\sqrt{T}$ and $\sqrt{R}$ respectively.

1. First, consider only two modes. The photon state in the input space can therefore be written $|1,1\rangle_{a,b}$. Give the single photon counting rates, $w_e^{(1)}$ and $w_f^{(1)}$ as a function of $\sqrt{T}$ and $\sqrt{R}$. Interpret this result.

2. Calculate the rate for counting one photon in each detector $w_{e,f}^{(2)}$.

3. What happens when $\sqrt{T} = \sqrt{R}$? One often says that this result is due to "two photon interference". What is actually interfering here?

   The remainder of the questions refer to the HOM paper. The authors take into account the fact that although slits can define (almost) single spatial modes, the individual frequencies of the photons coming from the source are not perfectly well defined and thus one must consider multiple modes with different frequencies for each spatial mode. They use the notation $|\omega_1\rangle = \hat{a}^\dagger_{\omega_1}|0\rangle$ to denote a state in which one frequency contains a single photon and all other frequencies are in the vacuum state. Equation 3 expresses the constraint that the sum of the two frequencies in the pair is well defined and equal to the pump frequency ω_0. They consider detection probabilities P_i which are proportional to our single photon rates $w_i^{(1)}$, and joint detection probabilities $P_{i,j}$ proportional to the joint count rates $w_{i,j}^{(2)}$.

4. If the experiment is ideal, the count rate at zero delay should vanish. One of the imperfections is the presence of "accidental coincidences". Give a reason why one might observe these accidental coincidences.

   The following is a way to estimate the accidental coincidence rate. Suppose that the count rate in each detector ($w^{(1)}$) is $2 \times 10^4\,\mathrm{s}^{-1}$, and that the coincidence count rate away from the dip $w^{(2)}(\delta\tau \neq 0)$ is $200\,\mathrm{s}^{-1}$. (These figures are not from the HOM paper but are approximately those of the version in the student lab at the Institut d'Optique.) If we neglect the correlation between the photons, what is the approximate rate at which one detects coincident counts in a 10 ns interval? How does this rate compare to the "true" coincidence rate away from the dip? If the above effect were the only imperfection in the experiment, what would be the contrast of the dip (the ratio between the coincidence rate middle of the dip

($\delta\tau = 0$) to that far from the dip). Why is it valid to estimate the accidental coincidence rate by neglecting the correlation between the photons?

5. The authors of the HOM paper subtracted these accidental coincidences from the plot in Fig. 2 of the paper. Nevertheless the dip does not go all the way to zero. What reason do the authors give for this observation?

6. Bonus question. Use the definition of a multimode field:

$$\hat{E}^{(+)}(t) = \sqrt{\frac{\hbar}{2\epsilon_0 V}} \sum_j \sqrt{\omega_j}\, \hat{a}_{\omega_j} e^{-i\omega_j t}$$

and the definition of a multimode two photon state given in Eq. 3 of the HOM paper to demonstrate Eq. 7 of that paper. Use our notation for the input and output ports: $(01, 02) \rightarrow (a, b)$ and $(1, 2) \rightarrow (e, f)$.

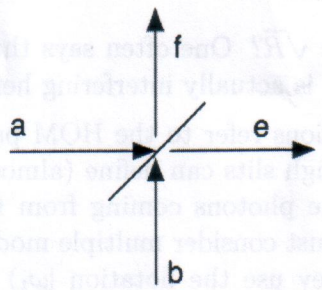

Figure 1: Beam splitter configuration in the HOM experiment.