

$4 \text{ gt } \sqrt{2}$ $\rightarrow 111 \rightarrow 2a$

use package {fourier}

Homework 1 - Black body radiation and photon quantum states

Due March 11th 2014

1 Mean photon number per mode

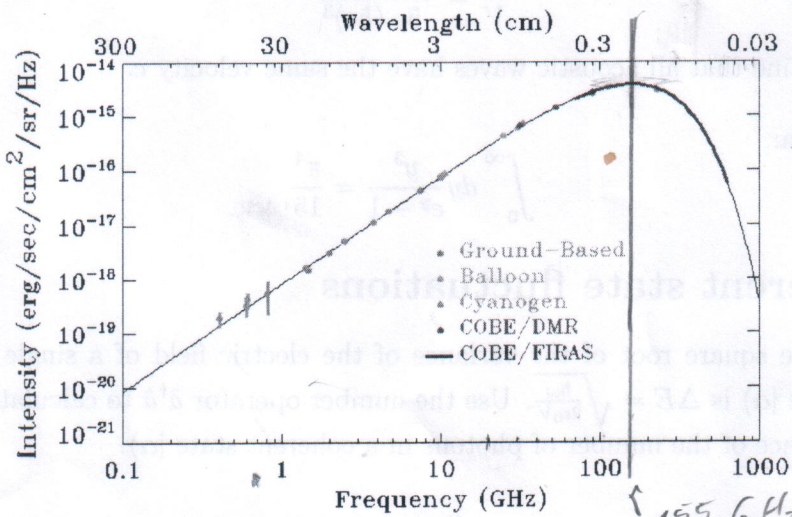
Calculate the mean number of photons per mode at the temperature of the sun (6000 K) in the visible part of the electromagnetic spectrum. $\rightarrow (400 \text{ } 800 \text{ nm})$
 $\hookrightarrow \text{integrate } n(\omega)$

2 Cosmic microwave background radiation

The figure below shows recent measurements of the energy density of the cosmic radiation. The quantity plotted is the spectral density of intensity, related to the density of energy per unit volume by $dI/d\omega = cu(\omega)/4\pi$ (Be careful with Hz and rad/sec!). Assuming Planck's law, estimate the temperature of the cosmic background.

Useful data: $1 \text{ erg} = 10^{-7} \text{ J}$, $k_B = 1.38 \times 10^{-23} \text{ J/K}$ et $c \approx 3 \times 10^8 \text{ m/s}$.

$u(\omega) = \frac{1}{\pi^2} \frac{\omega^2}{c^3} \frac{hc}{e^{k_B T} - 1}$
 $2.1c \rightarrow \times 10$



$\rightarrow u(\omega)$

$T = 2.64 \text{ K}$

$\uparrow 155 \text{ GHz}$

3 The Einstein and Debye theories of heat capacity

A solid can store energy in the vibrations of the atoms around their equilibrium positions. From the classical equipartition of energy, one predicts an energy $U = 3Nk_B T$ in a solid containing N atoms, and a heat capacity $C = \frac{dU}{dT} = 3Nk_B$. It was nonetheless known around 1900 that $C \rightarrow 0$ when $T \rightarrow 0$. We are going to understand this.

In 1907 Einstein applied the hypothesis of quantization proposed by Planck to the vibrations of the atoms, at frequency ω . If the vibrations are quantized, one should replace $k_B T$ by $\hbar\omega\bar{n}(T)$, with $\bar{n}(T)$ the mean occupation number of each oscillator at temperature T . This gives for the energy

$$U = \frac{3N\hbar\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1}$$

This trick allows C to decrease with temperature but it decreases too fast with respect to what was measured.

To do better, one notes that the vibrations in a solid don't all have the same frequency. One can consider the solid as a resonator for acoustic waves, in analogy with a resonator for electromagnetic waves. Use the argument on ~~mode counting~~ developed in the lecture and the hypothesis of quantization to show that the heat capacity per unit volume is given by Debye's formula

$$\frac{C}{V} = \frac{2\pi^2 k_B^4 T^3}{5 (\hbar c)^3}$$

$\rightarrow \times \hbar \omega \int dV$

You may assume that all acoustic waves have the same velocity c .

Useful formula:

$$\int_0^\infty dy \frac{y^3}{e^y - 1} = \frac{\pi^4}{15}$$

4 Coherent state fluctuations

Show that the square root of the variance of the electric field of a single mode in a coherent state $|\alpha\rangle$ is $\Delta E = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}$. Use the number operator $\hat{a}^\dagger \hat{a}$ to calculate the mean and the variance of the number of photons in a coherent state $|\alpha\rangle$.

} cons

5 Thermal state

Show that the variance of the number of photons in a single mode thermal field is $(\Delta n)^2 = \langle n \rangle^2 + \langle n \rangle$, with $\langle n \rangle$ the mean number of photons.