

## Research internship

### “Splines as universal solutions of L1-regularization problems”

#### Student:

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Note: This internship will constitute the 3rd trimester of my M.Sc.<sup>1</sup> in 2015–2016.

#### Subject<sup>2</sup>:

$L_1$ -regularization methods have shown to be very powerful for a large variety of image processing modalities as they tend to promote sparse solutions [5,6]. From a theoretical point of view, these methods exhibit strong links with the spline theory, in which a signal is described as a piecewise smooth function generated by a differential operator. This connection is well-understood for instance for the simple case of piecewise constant functions, which correspond to the derivative operator [5,7].

The goal of this project is to develop a theory for a more general class of operators. It will involve studying the conditions of invertibility of differential operators, with a particular emphasis on fractional operators.

We aim at strengthening the current framework [2], one goal being to develop a theorem extending the previous work done in the same team [3,4]. After an intense theoretical research phase, we plan to try to provide experimental evidence and examples of the results, in a way inspired from the current work the research group [1,3,4].

Keywords math. foundations of image processing; splines theory; wavelets;  $L_1$ -regularization.

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## Details of the internship

#### Period and duration:

- From 18th of April to 27th of August 2016, 4 months and a half.

#### Location:

- With the Bio-Medical Imaging Group (LIB), at EPFL, in Lausanne (Switzerland).

#### Supervision:

- Julien Fageot, [julien.fageot@epfl.ch](mailto:julien.fageot@epfl.ch), BM 4.139, Tel: +41 021 693 37 01 (Swiss) – 50%;
- Michael Unser, [michael.unser@epfl.ch](mailto:michael.unser@epfl.ch), BM 4.136, Tel: +41 021 693 51 75 (Swiss) – 50%.

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<sup>1</sup> The MVA, Master of Machine Learning, Vision and Applied Mathematics.

<sup>2</sup> This internship proposal was first offered on the BIG/LIB website, here [http://bigwww.epfl.ch/teaching/projects/subject.html#id\\_3734](http://bigwww.epfl.ch/teaching/projects/subject.html#id_3734).

**Internship tutor:**

- Gabriel Peyré<sup>3</sup>, [gabriel.peyre@ceremade.dauphine.fr](mailto:gabriel.peyre@ceremade.dauphine.fr), CEREMADE, Paris Dauphine University.

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**Bibliographic references****Main references:**

- [1] M. Unser & J. Fageot & J.P. Ward, *Splines are universal solutions of linear inverse problems with generalized TV-regularization*, December 2015 (under review).
- [2] T. Blu & M. Unser, *Self-Similarity: Part I – Splines and Operators*, IEEE Transactions on Signal Processing, vol.55, no.4, pp.1352-1363, April 2007, <http://bigwww.epfl.ch/publications/unser0701.html>.

**Other references:**

- [3] K.N. Chaudhury & M. Unser, *On the shift ability of dual-tree complex wavelet transforms*, IEEE Transactions on Signal Processing, vol.58, no.1, pp.221-232, January 2010, <http://bigwww.epfl.ch/publications/chaudhury0904.html>.
- [4] J.P. Ward & M. Unser, *Harmonic singular integrals and steerable wavelets in  $L_2(\mathbb{R})$* , Applied and Computational Harmonic Analysis, vol.36, no.2, pp.183-197, March 2014, <http://bigwww.epfl.ch/publications/ward1401.html>.
- [5] P. Tafti & M. Unser, *An introduction to sparse stochastic processes*, mainly chapter 5 (“Operators and their inverses”), Cambridge University Press, December 2013, <http://www.sparseprocesses.org/>.
- [6] S. Foucart & H. Rauhut, *A Mathematical Introduction to Compressive Sensing*, Springer, 2013, <http://link.springer.com/book/10.1007/978-0-8176-4948-7>.
- [7] S.D. Fisher & J.W. Jerome, *Splines solutions to  $L_1$ -extremal problems in one and several variables*, Journal of Approximation Theory 13.1 (1975): 73-83 (historical perspective).

Some of these research papers are previous work by the BIG team, cf. <http://bigwww.epfl.ch/publications/>.



Figure 1: Artist view of regular and fractional splines.

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<sup>3</sup> As required, he is a professor at the MVA master, he taught the “Sparsity and Compressive Sensing” course, for which I ranked first and got 19/20, with this small project <http://lbo.k.vu/pcs2016>.