

1. Let  $f(x, y) = (xy, y^2)$ . Evaluate  $\int_{C_i} f \cdot dr$  where  $C_i, i = 1, 2, 3$  are given below.
  - (a)  $C_1$  is the upper half of the unit circle traversing from  $(-1, 0)$  to  $(1, 0)$ ,
  - (b)  $C_2$  is the part of a curve  $y = x^2$  traversing from  $(0, 0)$  to  $(1, 1)$ ,
  - (c)  $C_3$  is the part of a curve  $y = x$  traversing from  $(0, 0)$  to  $(1, 1)$ .
2. Find the work done by the force  $f(x, y, z) = -xy \vec{i} + y^2 \vec{j} + z \vec{k}$  in moving a particle over a circular path  $x^2 + y^2 = 4, z = 0$  from the point  $(2, 0, 0)$  to the point  $(0, 2, 0)$ .
3. If  $f(x, y) = (2x + y^2) \vec{i} + (3y - 4x) \vec{j}$  evaluate  $\int_C f \cdot dr$  around a triangle  $ABC$  in the  $xy$ -plane with vertices  $A(0, 0), B(2, 0)$  and  $C(2, 1)$ .
  - (a) In the counter-clockwise direction.
  - (b) In the clockwise direction.
4. If  $f(x, y) = (y - x^2 e^x) \vec{i} + (\cos(2y^2) - x) \vec{j}$ , evaluate the line integral along the rectangle with vertices  $(0, 1), (1, 1), (1, 3)$  and  $(0, 3)$ .
5. A vector field is given by  $f(x, y) = \sin(y) i + x(1 + \cos(y)) j$ , evaluate the line integral over a circular path  $x^2 + y^2 = a^2, z = 0$ , where  $a \in \mathbb{R}$ . Also verify the result using Green's theorem.
6. Verify Green's theorem for  $f(x, y) = e^{-x} \sin(y) i + e^{-x} \cos(y) j$  and  $C$  is the square with vertices at  $(0, 0), (\pi/2, 0), (\pi/2, \pi/2)$  and  $(0, \pi/2)$ .
7. Find the work done by the force  $f(x, y) = (x^2 - y^3) i + (x + y) j$  in moving a particle along the path  $C$  bounded by the curves  $x + y = 0, x^2 + y^2 = 16$  and  $y = x$  in the first and fourth quadrants.
8. Using the Green's theorem evaluate the line integral  $\int_C ((2x - y^3) dx + xy dy)$  where  $C$  is the boundary of annulus region enclosed by the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 9$ .
9. Using Green's theorem prove that the area of the region  $D$  which bounded by a simple closed curve  $C$  can be found by  $\frac{1}{2} \int_C (x dy - y dx)$ . Then find the area of the ellipse.
10. Evaluate  $\int_C \frac{x dy - y dx}{x^2 + y^2}$  along any simple closed curve  $C$  in the  $xy$ - plane not passing through the origin. Distinguish the cases where the region  $D$  is enclosed by  $C$ :
  - (a) includes the origin,
  - (b) does not includes the origin.
11. Find the surface area of the cylinder bounded by  $x^2 + y^2 = a^2, z = 0$  and  $z = b, b > 0$ .
12. Find the surface area of the plane  $x + 2y + 2z = 12$  cut off by  $x = 0, y = 0$  and  $x^2 + y^2 = 16$ .
13. Evaluate  $\iint_S f \cdot n ds$ , where  $f = yz i + xz j + xy k$  and  $S$  is the part of the sphere  $x^2 + y^2 + z^2 = 1$  which lies in the first octant.
14. Evaluate  $\iint_S f \cdot n ds$ , where  $f = z i + x j - 3y^2 z k$  and  $S$  is the surface of the cylinder  $x^2 + y^2 = 16$  included in the first octant between  $z = 0$  and  $z = 5$ .

15. Let  $S$  denote the part of the plane  $2x + 5y + z = 10$  that lies inside the cylinder  $x^2 + y^2 = 9$ . Find the surface area considering  $S$  as a part of a graph  $z = f(x, y)$  where  $f(x, y) = 10 - 2x - 5y$ .
16. Find the area of the part of the surface  $z = x^2 + y^2$  that lies between the cylinders  $x^2 + y^2 = 4$  and  $x^2 + y^2 = 16$ .
17. Let  $S$  be the hemisphere  $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4, z \geq 0\}$ . Evaluate  $\iint_S z^2 \, ds$ , by considering  $S$  as a graph given by:  $z = f(x, y)$ .
18. Let  $S$  be a part of the cylinder  $y^2 + z^2 = 1$  that lies between the planes  $x = 0$  and  $x = 3$  in the first octant. Evaluate  $\iint_S (z + 2xy) \, ds$ .
19. Let  $\phi : \mathbb{R}^3 \rightarrow \mathbb{R}$ ,  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be twice differentiable and the second order partial derivatives are continuous on  $\mathbb{R}^3$ . Prove that
- (a)  $\overrightarrow{\text{curl}}(\overrightarrow{\text{grad}} \phi) = 0$  (b)  $\text{div}(\overrightarrow{\text{curl}} f) = 0$ .
20. Verify Stoke's theorem for  $f(x, y, z) = y \vec{i} + z \vec{j} + x \vec{k}$  where  $S$  is the upper half of the sphere  $x^2 + y^2 + z^2 = 1$  and  $C$  is its boundary.
21. Evaluate  $\int_C (2y^3 \, dx + x^3 \, dy + z \, dz)$  where  $C$  is the trace of the cone  $z = \sqrt{x^2 + y^2}$  intersected by the plane  $z = 4$  and  $S$  is the surface of the cone below  $z = 4$ .
22. Let  $S$  be the surface of the sphere  $x^2 + y^2 + z^2 = a^2$ . Show that  $\iint_S (\nabla \times V) \cdot n \, ds = 0$ , where  $V$  is any differentiable vector field in  $\mathbb{R}^3$ .
23. If  $f(x, y) = (2xy^2 + y) \vec{i} + (2x^2y + e^xy) \vec{j}$ , then find the condition on  $f$  such that it will be a conservative vector field. Then find the scalar potential function ( $\phi$ ) such that  $f = \nabla\phi$ .
24. Show that  $\int_C ((yz - 1) \, dx + (z + xz + z^2) \, dy + (y + xy + 2yz) \, dz)$  is independent of the path. Then evaluate the line integral from  $(1, 2, 2)$  to  $(2, 3, 4)$ .
25. Verify the Gauss divergence theorem for  $f = 2x^2y \vec{i} - y^2 \vec{j} + 4xz^2 \vec{k}$  taken over the region in the first octant bounded by the cylinder  $y^2 + z^2 = 9$  and the plane  $x = 2$ .
26. Let  $D$  be the region bounded by the hemisphere  $x^2 + y^2 + (z - 1)^2 = 9, 1 \leq z \leq 4$  and the plane  $z = 1$ . Let  $f(x, y, z) = x \vec{i} + y \vec{j} + (z - 1) \vec{k}$ , then verify Gauss divergence theorem.
27. Let  $D$  be the domain inside the cylinder  $x^2 + y^2 = 1$  cut off by the planes  $z = 0$  and  $z = x + 2$ . If  $f(x, y, z) = (x^2 + ye^z, y^2 + ze^x, z^2 + xe^y)$ , use divergence theorem to evaluate  $\iint_S f \cdot n \, ds$ .
28. Use the divergence theorem to evaluate the surface integral  $\iint_S f \cdot n \, ds$ , where  $f = (x + y, z^2, x^2)$  and  $S$  is the surface of the hemisphere  $x^2 + y^2 + z^2 = 1$  with  $z > 0$  and  $n$  is the outward unit normal to  $S$ .