NB : this document sums up some important techniques you should already be familiar with!

## Compute a finite sum or a finite product

We know how to compute finite sums and products:

- thanks to an inductive reasoning,
- thanks to a changing of index, or a telescoping,
- by using a geometric identity or the binomial theorem (for a sum).


## Changing of index

We can compute $\sum_{i=0}^{q-p} x_{p+i}$ by changing from $i$ to the new index $k:=p+i$ :
(1) we write $k=p+i$,
(2) we replace $i$ by $k-p$ in the sum,
(3) we precise the new bound for $k$ : when $i$ varies from 0 to $q-p, k$ varies from $p$ to $q$.

Exercise 1: Let $0 \leqslant p \leqslant n$, show that $\prod_{k=0}^{p-1} \frac{n-k}{p-k}=\binom{p}{n}$.

## Deal with inequalities

Bound a sum To upper-bound (resp. lower-bound, bound) the sum $\sum_{k=0}^{n} x_{k}$,
(1) for $k \in\{0, \ldots, n\}$, find an upper-bound (resp. lower-bound, bound) $M_{k}$ for the term $x_{k}$,
(2) then add each term in the right order: $\sum_{k=0}^{n} x_{k} \leqslant \sum_{k=0}^{n} M_{k}$.

Exercise 2: Prove: $\forall n \in \mathbb{N}^{*}, \frac{1}{n} \leqslant \sum_{k=0}^{n} \frac{1}{n^{2}+k^{2}} \leqslant \frac{n+1}{n^{2}}$.

Compute an integer part To compute an integer part, we usually come back to the definition, by bounding the real number $x$ by two consecutive integers. To conclude $\lceil x\rceil=n$, prove $n \leqslant x<n+1$.

Bound the absolute value of a sum We usually use on of the two triangular inequalities.

## Solve a polynomial equation

For an equation of degree lower (or equals) than 2, classical formulas can be used. For an equation of degree lower than 3 , we try to decrease the degree by a factorization (obvious root, remarkable identity $(a+b)^{2}=a^{2}+2 a b+b^{2}$ etc) or by a changing of unknown.

Exercise 3: In $\mathbb{R}$, solve the equation: $x^{3}-9 x+\frac{20}{x}$.

## Solve a linear equations system

We usually use the Gauss elimination method.

