

$$9) \int_4^{\infty} \frac{dx}{x^3 - 2x^2 - x + 2} = \int_4^{\infty} \frac{dx}{(x-2)[x^2-1]} = \int_4^{\infty} \frac{dx}{(x-2)(x-1)(x+1)}$$

$$\frac{1}{(x-2)(x-1)(x+1)} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$1 = A(x-1)(x+1) + B(x-2)(x+1) + C(x-2)(x-1)$$

$$1 = A(x^2-1) + B(x^2-x-2) + C(x^2-3x+2)$$

$$1 = (A+B+C)x^2 + (-B-3C)x + (-A-2B+2C)$$

$$\begin{aligned} A+B+C=0 \\ -B-3C=0 \\ -A-2B+2C=1 \end{aligned} \Rightarrow \begin{aligned} A-2C=0 \\ -A-2B+2C=1 \\ \hline -2B=1 \quad B=-1/2, \quad C=1/6, \quad A=-B-C \\ = 1/2 - 1/6 = 1/3 \end{aligned}$$

$$\Rightarrow \lim_{b \rightarrow \infty} \left[\frac{1}{3} \int_4^b \frac{dx}{x-2} - \frac{1}{2} \int_4^b \frac{dx}{x-1} + \frac{1}{6} \int_4^b \frac{dx}{x+1} \right]$$

$$\lim_{b \rightarrow \infty} \left[\frac{1}{3} \ln|x-2| \Big|_4^b - \frac{1}{2} \ln|x-1| \Big|_4^b + \frac{1}{6} \ln|x+1| \Big|_4^b \right]$$

$$\lim_{b \rightarrow \infty} \frac{1}{3} \ln|b-2| - \frac{1}{2} \ln|b-1| + \frac{1}{6} \ln|b+1| - \frac{1}{3} \ln(2) + \frac{1}{2} \ln(3) - \frac{1}{6} \ln(5)$$

$$\lim_{b \rightarrow \infty} \underbrace{\frac{\ln \left(\frac{(b-2)^{1/3} (b+1)^{1/6}}{(b-1)^{1/2}} \right)}{\rightarrow 1}} - \frac{1}{3} \ln(2) + \frac{1}{2} \ln(3) - \frac{1}{6} \ln(5)$$

Argument: dominants $\frac{b^{1/3} b^{1/6}}{b^{1/2}} = b^0 = 1 \Rightarrow \lim_{b \rightarrow \infty} \frac{(b-2)^{1/3} (b+1)^{1/6}}{(b-1)^{1/2}} = 1$

↳ on utilise la continuité de $\ln(x)$

$$\Rightarrow \dots = \frac{1}{3} \ln(2) - \frac{1}{2} \ln(3) + \frac{1}{6} \ln(5) \quad (\text{converge})$$

... Applications

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$$10) \quad M = k \int_0^{\infty} t e^{-kt} dt = k \lim_{b \rightarrow \infty} \int_0^b t e^{-kt} dt$$

$$\int_0^b t e^{-kt} dt \quad \text{par parties} \quad \begin{array}{l} u = t \\ du = dt \end{array} \quad \begin{array}{l} dv = e^{-kt} dt \\ v = \frac{-1}{k} e^{-kt} \end{array}$$

$$\begin{aligned} \int_0^b t e^{-kt} &= -\frac{t}{k} e^{-kt} \Big|_0^b + \frac{1}{k} \int_0^b e^{-kt} dt \\ &= -\frac{b}{k} e^{-kb} + 0 + \left(\frac{1}{k}\right) \left(\frac{-1}{k}\right) e^{-kt} \Big|_0^b \\ &= \frac{-b}{k} e^{-kb} - \frac{1}{k^2} e^{-kb} + \frac{1}{k^2} \end{aligned}$$

$$\Rightarrow k \lim_{b \rightarrow \infty} \left[\underbrace{\frac{-b}{k} e^{-kb}}_{\rightarrow 0} - \underbrace{\frac{1}{k^2} e^{-kb}}_{\rightarrow 0} + \frac{1}{k^2} \right] = k \left(\frac{1}{k^2} \right) = \frac{1}{k}$$