

Intégrales définies

$$1. \int_2^5 \cos(2\pi(x-2)) dx \quad (\text{par subs.})$$

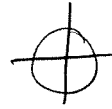
A) Avec subst. des bornes

$$u = 2\pi(x-2)$$

$$du = 2\pi dx$$

$$x_0 = 2 \Rightarrow u_0 = 2\pi(x_0 - 2) = 0$$

$$x_1 = 5 \Rightarrow u_1 = 2\pi(x_1 - 2) = 6\pi$$



$$\int_2^5 \cos(2\pi(x-2)) dx = \frac{1}{2\pi} \int_0^{6\pi} \cos(u) du = \frac{1}{2\pi} \sin(u) \Big|_0^{6\pi} = \frac{1}{2\pi} \underbrace{\sin 6\pi}_{=0} - \frac{1}{2\pi} \underbrace{\sin(0)}_{=0} = 0$$

B) Sans subs de bornes, on fait l'int. indéfinie d'abord

$$\int \cos(2\pi(x-2)) dx = \frac{1}{2\pi} \int \cos(u) du = \frac{1}{2\pi} \sin(u) = \frac{1}{2\pi} \sin(2\pi(x-2))$$

$$\int_2^5 \cos(2\pi(x-2)) dx = \frac{1}{2\pi} \sin(2\pi(x-2)) \Big|_2^5 = \frac{1}{2\pi} \underbrace{\sin(2\pi(5-2))}_{=0} - \frac{1}{2\pi} \underbrace{\sin(2\pi(0))}_{=0} = 0$$

$$2. \int_1^4 x \ln(\sqrt{x}) dx = \int_1^2 y^2 \ln(y) 2y dy = 2 \int_1^2 y^3 \ln(y) dy \quad \text{par parties}$$

$$y = x^{1/2}$$

$$dy = \frac{1}{2} x^{-1/2} dx$$

$$dx = 2x^{1/2} dy = 2y dy$$

$$x_0 = 1, u_0 = 1$$

$$x_1 = 4, u_1 = 2$$

$$2 \int_1^2 y^3 \ln(y) dy = 2 \left[\frac{y^4}{4} \ln(y) \right]_1^2 - 2 \int_1^2 \frac{1}{4} y^3 dy = \frac{1}{2} y^4 \ln(y) \Big|_1^2 - \frac{1}{2} \frac{y^4}{4} \Big|_1^2$$

$$= \frac{1}{2} (2)^4 \ln(2) - \frac{1}{2} (1)^4 \ln(1) - \frac{1}{8} (2)^4 + \frac{1}{8} (1)^4$$

$$= 8 \ln(2) - 1 + \frac{1}{8}$$

$$= 8 \ln(2) - \frac{7}{8}$$

$u = \ln(y)$	$dv = y^3$
$du = \frac{1}{y} dy$	$v = \frac{y^4}{4}$

$$3. \int_1^2 x^2 e^{2x} dx \quad (\text{voir dernier DGD : on sait que } \int x^2 e^{2x} dx =$$

$$\Rightarrow \int_1^2 x^2 e^{2x} dx =$$



ii) Suite) $f(x) \rightarrow$ soit \sim on \mathcal{L}

(3)

$$\left. \begin{array}{l} -2 < x < 0 : \text{ Prenons } x = -1 \\ f(-1) = (-1)^3 - 3(-1) + 3 = 5 \\ g(-1) = -1 + 3 = 2 \end{array} \right\} \Rightarrow f(x) > g(x) \text{ pour } -2 < x < 0$$

$$\left. \begin{array}{l} 0 < x < 2 : x = 1 \\ f(1) = 1^3 - 3 + 3 = 1 \\ g(1) = 4 \end{array} \right\} \Rightarrow f(x) < g(x) \text{ pour } 0 < x < 2$$

GRAPH

$$\begin{aligned} \text{Aire} &= \int_{-2}^2 |f(x) - g(x)| dx = \int_{-2}^0 f(x) - g(x) dx + \int_0^2 g(x) - f(x) dx \\ &= \int_{-2}^0 x^3 - 4x dx + \int_0^2 -x^3 + 4x dx \\ &= \left(\frac{x^4}{4} - 4 \frac{x^2}{2} \right)_{-2}^0 + \left(-\frac{x^4}{4} + 2x^2 \right)_0^2 \\ &= -\left(\frac{(-2)^4}{4} - 2(-2)^2 \right) + \left(-\frac{2^4}{4} + 2(2)^2 \right) \\ &= -(4 - 8) + (-4 + 8) \\ &= -8 + 16 = 8 \end{aligned}$$

Applications Barre verticale \int_0^{110}
110 cm

Densité linéique: $p(z) = e^{-0,001z}$ $\left[\frac{g}{cm} \right]$

Masse totale: $M = \int_0^{110} e^{-0,001z} dz$

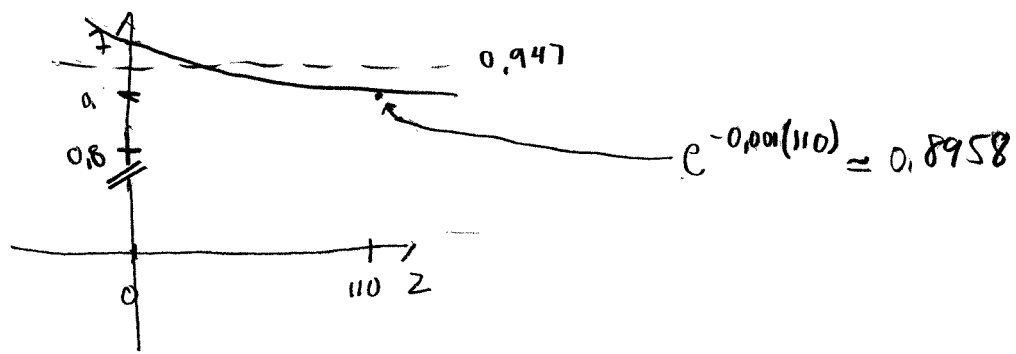
$$\begin{aligned} &= \int_{-0,11}^0 e^u du \\ &= -1000 e^u \Big|_0^{-0,11} = -1000 (e^{-0,11} - \underbrace{e^0}_{=1}) = -1000 (e^{-0,11} - 1) \approx 104,17 \end{aligned}$$

$u = -0,001z$ $u_0 = 0$
 $du = -0,001 dz$ $u_1 = -0,001 \cdot 110 = -0,11$

Densité moyenne: $\frac{1}{110} \int_0^{110} e^{-0,001z} dz = \frac{-1000 (e^{-0,11} - 1)}{110} \approx 0,947$

$\frac{1}{z_1 - z_0}$

Graphique



Applications (débit d'eau)

débit: $1 - e^{-t}$ $\left[\frac{\text{cm}^3}{\text{s}} \right]$ entre $t=0$ et $t=2$

Qte totale d'eau écoulée de ce temps $[\text{cm}^3]$

$$Q = \int_0^2 (1 - e^{-t}) dt = t - e^{-t} \Big|_0^2 = 2 - e^{-2} - (0 - \underline{\underline{1}}) = 3 - e^{-2} \approx$$

Débit moyen $\frac{1}{t_1 - t_0} \int_0^2 (1 - e^{-t}) dt = \frac{1}{2} (3 - e^{-2}) \approx$

Q: Quelle qte d'eau si l'eau coule au débit moyen?

$$Q = \int_0^2 \frac{1}{2} (3 - e^{-2}) dt = \frac{1}{2} (3 - e^{-2}) t \Big|_0^2 = (3 - e^{-2})$$

Applications Réaction chimique $\frac{dC}{dt} = t e^{-t}$

À $t=0$, aucun produit C $\Leftrightarrow C(0) = 0$

Quelle qte à $t=2$.

$$Q_{te} = \int_0^2 \frac{dC}{dt} dt = \int_0^2 t e^{-t} dt$$

par parties $\int u dv = uv - \int v du$

$u = t$	$dv = e^{-t}$	$u_0 = 0$
$du = dt$	$v = -e^{-t}$	$u_1 = 2$

$$= -t e^{-t} \Big|_0^2 + \int_0^2 e^{-t} dt = -t e^{-t} \Big|_0^2 - e^{-t} \Big|_0^2$$

$$\dots = \underbrace{[-2e^{-2} - e^{-2}]}_{-3e^{-2}} - \underbrace{[0 - e^0]}_{+1} = -3e^{-2} + 1 \approx 0,594$$

Comment utiliser que $(0) = 0$

Qte totale = Qte initiale + qte produite entre $t=0$ et $t=2$

$$\Rightarrow \text{Qte totale} = -3e^{-2} + 1$$

Applications Nb d'individus: densité $f(x) = 3 \cdot 10^{-4} x(1000-x)$ $0 < x < 1000$

$$\begin{aligned} \text{Nb total} &= \int_0^{1000} 3 \cdot 10^{-4} x(1000-x) dx = 3 \cdot 10^{-4} \int_0^{1000} (1000x - x^2) dx \\ &= 3 \cdot 10^{-4} \left(1000 \frac{x^2}{2} - \frac{x^3}{3} \right)_0^{1000} \\ &= 3 \cdot 10^{-4} \left(500 \cdot 1000^2 - \frac{1}{3} (1000)^3 \right) \\ &= 3 \cdot 10^{-4} \left(5 \cdot 10^8 - \frac{1}{3} \cdot 10^9 \right) \\ &= 15 \cdot 10^4 - 1 \cdot 10^5 \\ &= 1.5 \cdot 10^5 - 1 \cdot 10^5 \\ &= 5 \cdot 10^4 \text{ loutres} \end{aligned}$$

Densité moyenne: $\frac{1}{1000} \cdot 5 \cdot 10^4 = 5 \left[\frac{\text{loutres}}{\text{mile}} \right]$